Page 1

## IMPEDANCE TESTS OF SINGLE OR COUPLED RESONATORS by Harold A. Wheeler

A line is terminated in a single resonator or a coupled pair of resonators. The terminal impedance locus is measured by SWR in the line and is plotted on a reflection chart. Certain points on the plot are identified and are used to compute the power factors of the resonators and of the line loading, and the coefficient of coupling. Any one of these may be found directly by observing certain frequencies on the locus.

 $p_1$  = (unloaded) power factor of primary resonator

p1' = loading power factor of primary resonator

 $p_1$ " =  $p_1$ ' +  $p_1$  = loaded power factor of primary resonator

p<sub>2</sub> = power factor of secondary resonator

k = coefficient of coupling between primary and secondary resonators

k<sub>X</sub> = apparent coefficient of coupling at crossover of loop in locus

g = normalized conductance (scale on axis of chart)

Bach pair of points marked on a locus corresponds to a pair of frequencies  $(f_+\,,\,f_-)$  such that

the indicated p or 
$$k = \frac{f_+ - f_-}{f_0}$$
 (1)

Relations for coupled circuits: (Fig. 3)

$$k^2 = k_x^2 + p_2^2; (2)$$

$$(p_2/k)^2 = \frac{g_x - g_1}{g_0 - g_1}$$
;  $(k_x/k)^2 = \frac{g_0 - g_x}{g_0 - g_1}$  (3) (4)

Determine p<sub>2</sub> directly by upper dotted circle.

Determine k directly by lower dotted circle, if there is a loop and crossover:  $p_2^2 < k^2$ .

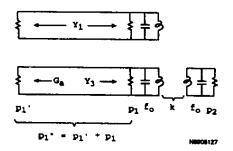


Fig. 1 - The circuit of a line and single or coupled resonators.

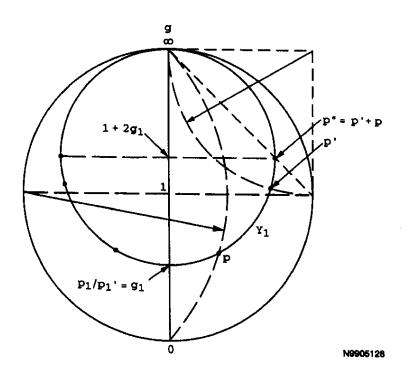


Fig. 2 - The reflection chart of single resonator.

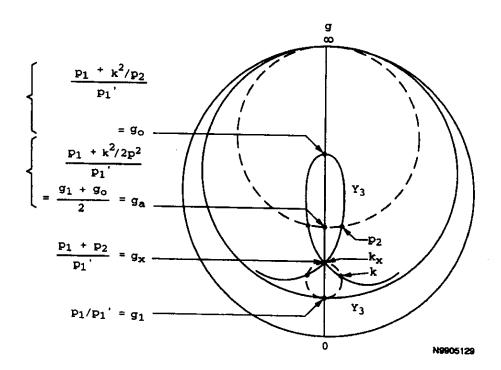


Fig. 3 - The reflection chart of coupled resonators.

Report 665

Wheeler Laboratories, Inc.

Page 4 Revised 560605

Preferred procedures are as follows.

Weak coupling (no loop):  $k^2 < p_2^2$ . Compute  $p_1$ ' for primary.

Note  $g_1$  and  $g_0$ ; draw dotted circle through  $g_a$ ; compute  $p_2$  by (1);

$$k = \sqrt{P_1' P_2(g_0 - g_1)}$$
 (5)

Moderate coupling (small loop):  $p_2^2 < k^2 < 2p_2^2$  Evaluate  $p_2$  as above.

Draw dotted circle through  $g_1$  and  $g_\chi$ ; compute k by (1).

Strong coupling (large loop):  $2p_2^2 < k^2$ Compute  $p_1$ ' for primary. Note  $g_1$  and  $g_x$ ;

$$p_2 = p_1'(g_x - g_1)$$
 (6)

At crossover, compute  $k_{\chi}$  by (1);

$$k = \sqrt{k_x^2 + p_2^2} = k_x \sqrt{\frac{g_0 - g_1}{g_0 - g_x}}$$
 (7)

The latter formula for k does not require  $p_1$ ' but does require  $g_0$ .

Instead of computing  $p_2$  before k, the following procedure gives  $p_2$  after k; it is most useful if  $p_2$  is so small it is difficult to measure directly: After determining k by lower dotted circle,

$$p_2 = \frac{k^2}{p_1'(g_0 - g_1)} \tag{8}$$

50124

NB 68, p. 27-34. Also see J-323.

J-102

ew

Hawheeler