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FROM: H. A. Wheeler

SUBJECT: Synthesis of a double-tuned wideband matching network for a small antenna.

Abstract. An orderly procedure is given for the synthesis of a double-tuned wideband matching network for a small antenna. It is based on a known dummy antenna, such as that described in RL8190 for a disc-loaded monopole that is moderately "small". This problem is characterized by asymmetric behavior over the band, so it is not susceptible to simple treatment based on symmetry. The result is believed to be optimum (or very nearly so) by comparison with the known analytic optimum for a ratio-symmetric wideband model.



[AP-2] A logarithmic reflection chart for presentation of antenna impedance, 800801.
[RL8168] The design of a wideband matching network for a small antenna, 810601.
[RL8190] Circuit representation of the radiation from a disc-loaded monopole, 820503.



Over a specified frequency band, a small antenna is to be matched with a long line represented by a constant resistance ($R_o$). See Fig. 1. The antenna is represented by a dummy made of a few constant circuit elements. An interconnecting network configuration of lossless fixed reactors is described for wideband matching. In that network, there is a set of coefficients which will yield "maximin" efficiency of matching or "minimax" reflection VSWR (S) over the band. The procedure to be described is capable of approaching this "optimum" result in a series of simple steps.

The simplest wideband matching has been characterized by ratio symmetry so it could be identified with a low-pass analog. With double-tuning, the optimum is achieved by presenting equal pure resistance of one extreme value at both cutoff frequencies and pure resistance of the opposite extreme value at an intermediate frequency. The simplicity of the low-pass analog enables a simple analytic proof that this pattern is the optimum.

The radiation loading of a small antenna varies with frequency in such a manner that the matching departs from wideband ratio symmetry. The same rule can be applied to give equal pure resistance at both cutoff frequencies. However, the extreme impedance at "midband" is not pure resistance and the locus on the hemisphere chart departs substantially from symmetry. In the example to be described, near-symmetry is attained by adding a series capacitor. It has only a minor effect at the cutoff frequencies and even that can be compensated by a minor adjustment. The result is similar to the symmetrical matching. It is believed to be likewise optimum or very nearly so.

The small antenna is here exemplified by a disc-loaded monopole, which is the subject of RL8190. Its external field of radiation is represented as described therein.

Fig. 1 shows the network configuration. The dummy antenna has two parts:

- $R_1L_1C_1$ representing the exterior field of radiation (as in RL8190);
- $C_2$ representing some interior shunt C inherent under the outer area of the disc.

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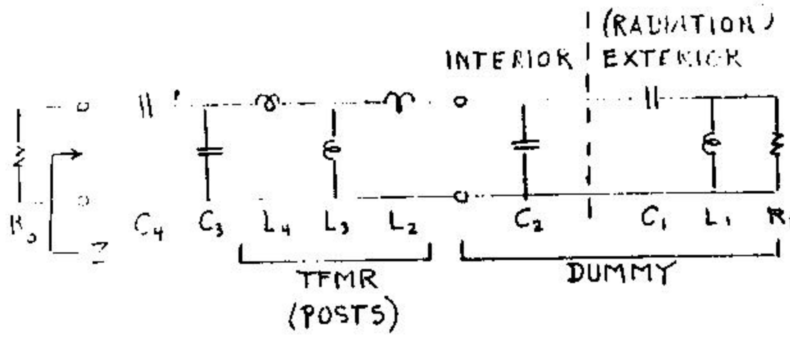


Fig. 1 - The dummy antenna and the matching network for double tuning.

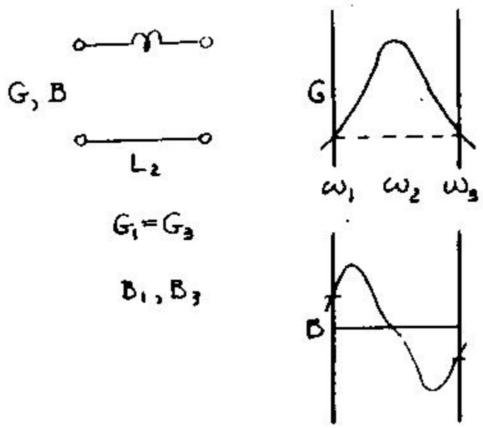


Fig. 2 - The midband series tuning of the dummy antenna.



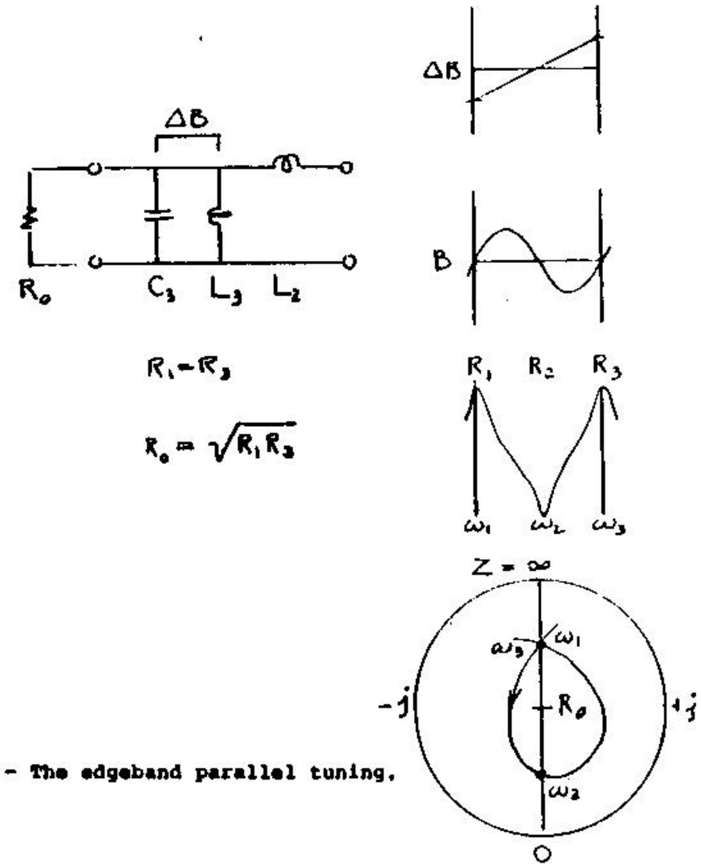


Fig. 3 - The edgeband parallel tuning.

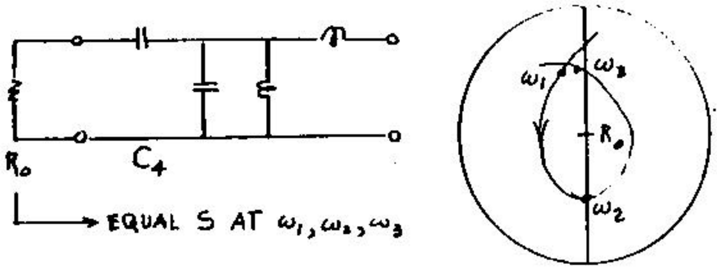


Fig. 4 - The series capacitor for reducing asymmetry.

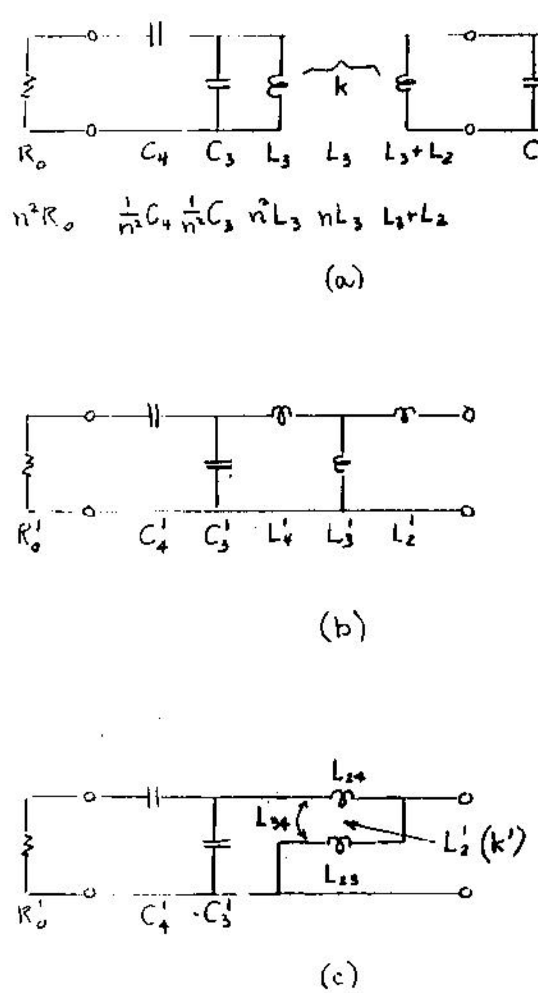


Fig. 5 - The transformer.



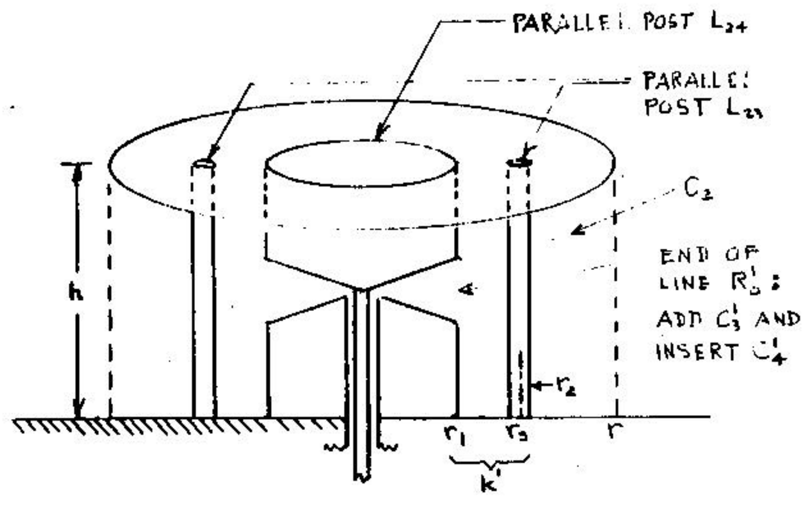


Fig. 6 - Embodiment in a disc-loaded monopole.

The latter is less than that under the entire area, and is taken to be 1/2. The remaining network of lossless reactors provides for double tuning, including the dummy. The line is represented by $R_o$. The triplet of inductors ($L_2L_3L_4$) can be used as a transformer for adapting to any $R_o$ within a range. From the few options for double tuning, the configuration is chosen for these features:

- To tolerate more shunt C ($C_3$);
- To match greater $R_o$, which happens to be desired for this example.

Both series and shunt L are required for closest matching of the (mainly capacitive) dummy. As will be seen, all coefficients in the matching network are uniquely determined by the set of conditions to be satisfied.

For convenience in numerical analysis, the values may be normalized to a set of units such that all values are not far from unity:

- Since the lower cutoff is particularly significant in matching a small antenna, take $\omega_1 = 1$.
- At this frequency, take the reactance of the total C of the disc to be 1.

These determine the 4 essential units, as in RL8190.

On the first run, leave $R_o$ unspecified and omit $L_4$ (=0). Later, specify $R_o$ to determine $L_4$. The procedure is to be described in the order of Figs. 2-5. The only measurements (by computation) refer to the input impedance presented to $R_o$:

$$Z = 1/Y; \; Z = R + jX; \; Y = G + jB$$

Referring to Fig. 2, for preliminary series tuning, set $L_2$ by trial to give equal G at lower and upper cutoff ($\omega_1$ and $\omega_3$):

$$G_1 = G_3$$

Note the corresponding values of susceptance ($B_1$ and $B_3$).

Referring to Fig. 3, compute $C_3$ and $L_3$ explicitly to cancel $B_1$ and $B_3$ by $\Delta B$. The result is equal peaks of $|Z|$ which are pure R at cutoff, and a valley near midband. On the hemisphere chart, the ends of the locus coincide at the top ($R_1=R_3$). The midband $\omega_2$ is defined for pure $R_2$ at the bottom. A preliminary value of $R_o$ for near-minimum S is taken as the mean:

$$R_o = \sqrt{R_1R_2}$$

The loop departs from symmetry, such that S is greater above midband.



Referring to Fig. 4, a series capacitor $C_4$ is inserted to reduce the asymmetry of the loop. Its value is set by trial until:

- At the new value of $\omega_2$ for pure $R_2$;
- The S is at a maximum (which means zero slope of S near $\omega_2$).

The result is near-symmetry of the loop.

However, the series $C_4$ causes an unequal shift of the upper ends of the loop, slightly to the left. If justified, this small shift can be compensated by a convergent repetition of these operations:

- Adjust $C_3$ and $L_3$ to nullify the susceptance $B_1$ and $B_3$. This will leave a slight separation of $G_1$ and $G_3$.
- Adjust $L_2$ to equalize $G_1$ and $G_3$. This will leave a slight separation of $B_1$ and $B_3$.

The result of convergence will be equal pure $R_1$ and $R_3$ while retaining near-symmetry.

With or without this refinement, a final value of $R_o$ is determined by trial to equalize max $S_2$ (near midband) with the greater of $S_1$ and $S_3$ (at edgeband). It will be near the mean formulated above. The resulting $S_2$ is the minimax (to a close approximation).

So far, $R_o$ has been one of the 5 variables needed to achieve the minimax S obtainable with this double tuning of this dummy antenna.

Referring to Fig. 5, a transformation canbe made to give this result with a specified value of $R_o$.

In Fig. 5(a), series $L_2$ and shunt $L_3$ are shown as an equivalent transformer with coefficient of coupling:

$$k = \sqrt{L_3/(L_3+L_2)} \qquad (1)$$

The lower row of coefficients shows the impedance on the left side (including $R_o$) multiplied by $n^2$.

In Fig. 5(b), the coefficients in the lower row are renamed and the transformer is represented by a triplet of uncoupled inductors (as in Fig. 1):

$L_2' = L_2 + L_3 - nL_3 = L_2 - (n-1)L_3$
$L_3' = nL_3$
$L_4' = n^2L_3 - nL_3 = n(n-1)L_3$

$$n = \sqrt{R_o'/R_o} \qquad (2)$$

As will be seen, the triplet is to be made of two coupled inductors, one in series and one in parallel with the dummy antenna. Fig. 5(c) shows this configuration equivalent to the triplet:

$L_{23} = L_2' + L_3'$
$L_{24} = L_2' + L_4'$

$$k' = L_2'/\sqrt{L_{23}L_{24}} \qquad (3)$$

The series L of the two inductors is:

$$L_{34} = L_3' + L_4' = n^2L_3 \qquad (4)$$

This will be found convenient to measure.

Fig. 6 shows the structure of a disc-loaded monopole which can be designed to realize the computed network. The exterior radiation loading corresponds to that developed in RL8190, here represented by $R_1L_1C_1$. The interior network comprises these components:

$C_2$ = inherent C under the outer 1/2 of the disc area;
$L_{23}$ = parallel L of a pair of vertical wires;
$L_{24}$ = series L of a center post;
$k'$ = coupling therebetween;
$C_3'$ = inherent and added C in parallel;
$C_4'$ = inserted C in series.

The line is extended by a biconical section to the outer wall of the center post, where it presents $R_o'$ at the gap. Parallel C can be added by a "capacitive iris". Series C can be built inside the post.



Structural realization of the design is complicated by the mixed fields under the disc, so C or L cannot be measured individually. Placing the end of the line in a gap half-way up the center post has the effect of minimizing the interaction of fields.

The one quantity that can be measured individually is $L_{34}$, the series L of the post and wires with their coupling. It can be measured on the L-meter at about 1 MHz. It reflects the coupling therebetween.

First the radius ($r_1$) of the center post can be adjusted approximately by this procedure:

- Remove the wires ($L_{23}$) and omit $C_3'$ and $C_4'$.
- Compute the series-resonance frequency ($\omega_{24}$) of $L_{24}$ with the dummy.
- Measure this frequency of minimum Z or zero R. The separation between these two conditions indicates the uncertainty of defining this frequency.
- Adjust the radius to give a measured frequency close to the computed frequency.

From $r_1$ and $L_2'$ can be computed the required radius ($r_3$) to the wires:

$$L_2' = \mu_o h(1/2\pi)\ln r_3/r_1$$

$$r_3 = r_1 \exp(2\pi L_2'/h\mu_o) \qquad (5)$$

From these radii and $L_{34}$ can be computed the required radius ($r_2$) of each wire. In the examples studied to date, the center-post radius is large enough to shield the wires from each other, so each wire can be computed individually. Also each wire is small enough to use the small-wire approximation. Then this formula can be used for each wire:

$$2L_{34} = \mu_o h(1/2\pi)\ln[2(r_3-r_1)(1/r_2 + 1/r_1)]$$

$$1/r_2 = \frac{1}{2(r_3-r_1)} \exp(4\pi L_{34}/h\mu_o) - 1/r_1 \qquad (6)$$

A structure with these dimensions can be measured before providing parallel and series C. Its locus on the chart would indicate what should be added to $C_3'$ so that the insertion of computed $C_4'$ will yield the desired closed loop. It should be centered near the specified $R_o'$ and should give max S not much greater than the minimax predicted by computation.

To the extent that the structure may fall short of prediction, some trimming of the dimensions under the disc may be required. The effects of the parts are not simple, so no formula for the trimming process is apparent.

## Formulas for network computation.

The indicated computations are familiar but these formulas may be helpful in programing, as by subroutines.

The input Z of the ladder network is computed from right to left, stopping after any number of arms. Each step requires inversion and the addition of either:

- jX in series with Z or
- jB in parallel with Y=1/Z.

The result is reduced to Z by an extra inversion if necessary.

This formula for inversion has been found helpful as a subroutine, for its simplicity and its time saving by avoiding trigonometric functions:

$$G = \frac{R}{R^2+X^2}\ ;\ B = \frac{-X}{R^2+X^2} \tag{7}$$

Or vice versa:

$$R = \frac{G}{G^2+B^2}\ ,\ X = \frac{-B}{G^2+B^2} \tag{8}$$

The same subroutine can be used for R,X → G,B or vice versa. The HP stack of 4 registers (and last X) enables this subroutine without any other storage registers.



From $R_o$ and Z=R+jX, the VSWR can be computed in two steps:

$$\rho^2 = 1 - \frac{4R_oR}{(R_o+R)^2 + X^2} \tag{9}$$

$$S = \frac{1+\rho}{1-\rho} = 1 + \frac{2}{1/\rho - 1} \tag{10}$$

One step in the synthesis (Fig. 3) requires the cancellation of $B_1$ and $B_3$ at $\omega_1$ and $\omega_3$ by adding the parallel $L_3$, $C_3$. This subroutine gives an explicit solution for the latter to provide $-B_1$ and $-B_3$:

$$C_3 = \frac{B_1\omega_1 - B_3\omega_3}{\omega_3^2 - \omega_1^2} = \frac{B_1/\omega_3 - B_3/\omega_1}{\omega_3/\omega_1 - \omega_1/\omega_3} \tag{11}$$

$$1/L_3 = \frac{B_1\omega_3 - B_3\omega_1}{\omega_3^2 - \omega_1^2}\,\omega_1\omega_3 = \frac{B_1\omega_3 - B_3\omega_1}{\omega_3/\omega_1 - \omega_1/\omega_3} \tag{12}$$

The rectangular coordinates of the logarithmic reflection chart have been found useful in reporting the computed Z. They offer the features described in memo AP-2. The common log is convenient.

$$z = \log|Z| = \tfrac{1}{2}\log(R^2 + X^2) \tag{13}$$

$$x = \log S_x = \log\left[X/R + \sqrt{1+(X/R)^2}\right] = \text{antisinh}_{10} X/R \tag{14}$$

Unlike the hemisphere chart, these coordinates do not depend on the choice of a reference $R_o$, so they can be reported without waiting for that choice.

The print-out on the tape of a small calculator might contain any of these groups:

| | |
|---|---|
| $\omega$ | $\omega_1$ |
| x | $\omega_3$ |
| z | $B_1$ |
| X | $B_3$ |
| R | $C_3$ |
| B | $L_3$ |
| G | |
| $R_o$ | |
| S | |