TABLE I
"IDEAL" FEED FOR EACH TYPE OF "CONIC" REFLECTOR

| Conic Surface | Eccentricity | Feed |
|---|---|---|
| Sphere | $\epsilon=0$ | Magnetic Dipole (Electric Dipole=0) |
| Ellipsoid | $0<\epsilon<1$ | Electric+Magnetic Dipole ($ED=\epsilon\ MD$) |
| Paraboloid | $\epsilon=1$ | Electric+Magnetic Dipole ($ED=MD$) |
| Hyperboloid | $1<\epsilon<\infty$ | Electric+Magnetic Dipole ($ED=\epsilon\ MD$) |
| Plane | $\epsilon=\infty$ | Electric Dipole (Magnetic Dipole=0) |

polarization characteristics are expressed in terms of the polarization characteristics of elemental electric and magnetic dipole radiators in the proper intensity ratio.

ACKNOWLEDGMENT

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# The Geometrical Theory of Diffraction Applied to Antenna Pattern and Impedance Calculations

ALFRED R. LOPEZ, MEMBER, IEEE

*Abstract*—The geometrical theory of diffraction is applied to the calculation of the radiation pattern and impedance of a monopole antenna on a perfectly conducting circular ground plane of limited extent. In this calculation, the radiation problem is resolved into two components, one being the monopole contribution and one the edge contribution. The impedance problem is resolved into the components of a reflection from the monopole in an infinite ground plane and a reflection from the circular edge as seen through the antenna. The known solutions of these individual components then permit the calculation of the overall radiation pattern and impedance by superposition. The techniques described are general and are considered applicable to a large class of similar radiating structures.

## I. INTRODUCTION

SINCE THE introduction [1] of the geometrical theory of diffraction in the early 1950's, it has been employed successfully in the solution of various types of diffraction problems. In this paper, the theory is applied to the problem of determining the radiation pattern and impedance of a monopole antenna operating over a finite perfectly-conducting circular ground plane (disk-monopole antenna). This configuration is shown in Fig. 1. The purpose of this present paper is not primarily to solve this particular problem, but rather to demonstrate the usefulness of the geometrical theory of diffraction in the solution of a class of radiation problems, which arise often in practice.

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Simply stated, the geometrical theory of diffraction proposes that the total electromagnetic field, excited by sources, can be described in terms of a superposition of rays emanating from the sources and rays diffracted from edges or other diffracting obstacles. The essential feature of the theory is that it is applicable to all types of complicated problems, some of which are, in practice, impossible to solve rigorously. The approach is to resolve a complicated problem into simpler component problems, each of which have relatively simple rigorous solutions. In the practical application of the theory, there are available many known rigorous solutions of diffraction problems (e.g., wedge, cone, cylinder, sphere, etc.) which then serve as basic building blocks for the construction of the solutions for the more complicated problems.

One such problem is the determination of the characteristics of a disk-monopole antenna. In this paper, the radiation pattern and impedance of the monopole configuration will be determined by means of the geometrical theory of diffraction. The pattern and impedance will be constructed from the rigorous solutions of a monopole radiating in an infinite ground plane and the diffraction by a straightedge excited by a magnetic line

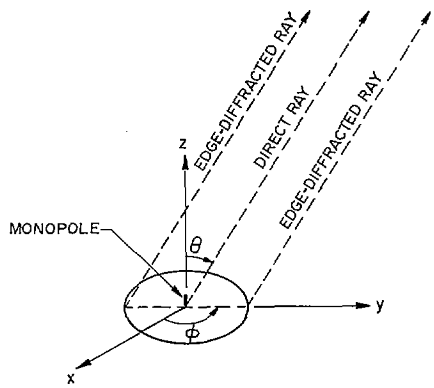



Fig. 1. Monopole on a circular ground plane.

source, since the solution to the latter problem is readily available. (The magnetic line source and the monopole produce the same fields in the near neighborhood of the edge as the source-to-edge distance approaches infinity. If this distance is large in terms of wavelengths, the fields in the edge neighborhoods, and, therefore, the diffracted fields, are approximately the same for the two cases.)

The pattern calculation will consist of a superposition of a ray emanating from the monopole itself and two rays emanating from the edge of the circular ground plane. In a similar manner, the impedance calculation will be a superposition of a reflection caused by the monopole operating in an infinite ground plane and a reflection caused by the finite circular ground plane. The magnitude and phase of the edge reflection will be determined by utilizing some common radar principles.

The diffraction problem of a straightedge excited by a magnetic line source will be reviewed, since it is essential to the discussion. The radiation pattern of the disk-monopole antenna will then be calculated and compared with experimental results. The impedance of the same configuration will be calculated and compared to a result found in the literature.

## II. List of Symbols

- $\rho$ = distance from the observation point to the straightedge.
- $\rho'$ = distance from the source point to the straightedge.
- $\beta$ = angle measured into the shadow region from the shadow boundary with the origin at the straightedge.
- $k$ = free-space propagation constant.
- $\lambda$ = free-space wavelength.
- $E(\beta)$ = ratio in the far field of the diffracted field to the value of the incident (geometrical optics) fields at the shadow boundary.
- $\beta_{3\text{ dB}}$ = angle at which diffracted signal is down 3 dB from its shadow boundary value.
- $\theta$ = antenna pattern angle.
- $\phi$ = angle measured along the ground plane.
- $P_R$ = received power.
- $P_T$ = transmitted power.
- $G$ = antenna power gain in direction of the edges.
- $V_R$ = received voltage.
- $V_T$ = transmitted voltage.
- $V_i$ = incident voltage in transmission line.
- $V_r$ = reflected voltage in transmission line.
- $\Gamma$ = antenna reflection coefficient.
- $\Gamma_\infty$ = antenna reflection coefficient for an infinite ground plane.
- $\Gamma_e$ = antenna reflection coefficient component attributed to edge.
- $\tau_\infty$ = antenna transmission coefficient for an infinite ground plane.
- $Z$ = antenna impedance.
- $Z_\infty$ = antenna impedance for an infinite ground plane.
- $Z_e$ = antenna impedance component attributed to edge.
- $Z_0$ = characteristic impedance of transmission line.
- $h$ = height of monopole.
- $I(x)$ = current-distribution function of monopole.
- $I(o)$ = base current or input current.

## III. Far-Field Diffraction Pattern of an Edge

As indicated in Section I, the problem of determining the far-field pattern of a monopole radiating in a finite circular ground plane can be constructed from the rigorous solutions of a monopole radiating in an infinite ground plane and the diffraction by a straightedge excited by a magnetic line source. In this section, the straightedge problem is reviewed. The details of the solution can be found in the literature [8]; for our purpose it is sufficient to summarize the pertinent results.

Figure 2 is a sketch showing the magnetic line source and the diffracting edge. If, in the general solution of this problem, the far-field condition ($k\rho \gg k\rho'$) is imposed, the ground plane is large in terms of wavelengths ($k\rho' \gg 1$), and the time dependence is exp ($j\omega t$), then the far-field diffraction pattern $E(\beta)$ is related to a complex Fresnel integral and may be summarized as follows:

$$E(0) = \frac{1}{2} \tag{1}$$

$$E(\pm\beta_{3\text{ db}}) = \pm\frac{1}{2\sqrt{2}}\exp(-j\,0.095\pi) \tag{2}$$

$$\beta_{3\text{ db}} = \pm\, 2\arcsin\left(\frac{0.3}{\sqrt{k\rho'}}\right) \tag{3}$$

$$E(\beta) = \frac{1}{2}\,\frac{\csc\left(\dfrac{\beta}{2}\right)}{\sqrt{2\pi k\rho'}}\exp\left(-j\frac{\pi}{4}\right). \tag{4}$$

This last expression is asymptotic to the Fresnel integral and is a good approximation when

$$|\beta| \geq 2\arcsin\left(\frac{2}{\sqrt{k\rho'}}\right). \tag{5}$$

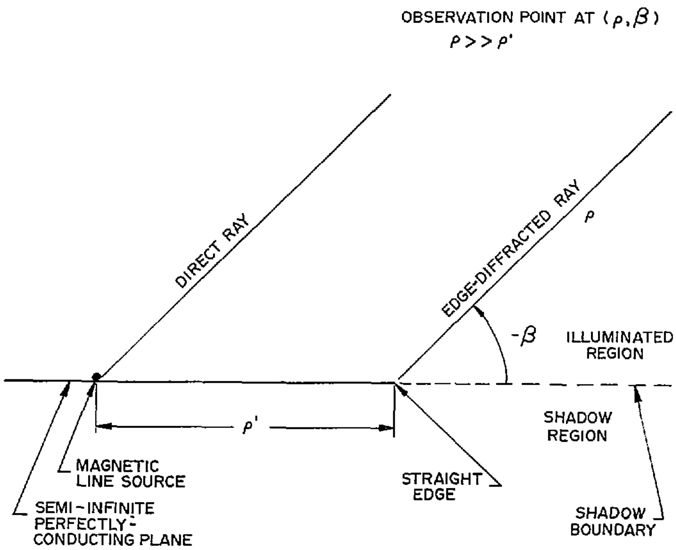



Fig. 2. Magnetic line source excitation of a straightedge.

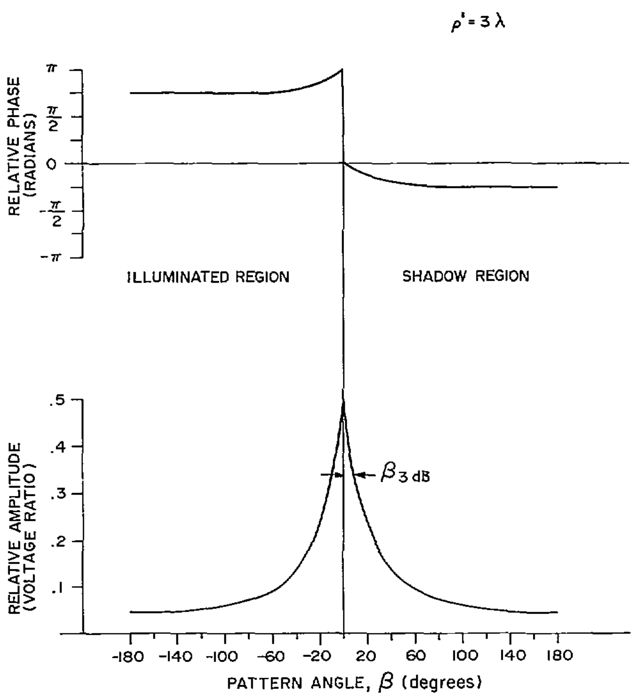



Fig. 3. Far-field diffracted-signal patterns of a straightedge excited by a magnetic line source.

Figure 3 shows the amplitude and phase patterns of the radiation pattern as indicated by (1), (2), and (4) for the case where $\rho' = 3\lambda$. Note the peak in the amplitude pattern and the discontinuity in the phase pattern at the shadow boundary. As is expected for this problem, the discontinuity in the phase pattern of the diffracted signal is precisely what is required to counteract the amplitude discontinuity in the direct signal (geometrical optics field).

## IV. Radiation Pattern of a Monopole above a Finite Circular Ground Plane

The calculation of the radiation pattern for the configuration under consideration can now be completed using the notions of the geometrical theory of diffraction. It follows that the radiation pattern in any plane containing the monopole consists of the monopole pattern plus the patterns of the edge-diffracted signals when added in the proper relative phase. In this case, the characteristics of the edge-diffracted signals in the plane of interest are as described in Section III for the case of the two-dimensional straightedge problem.

Figure 1 shows the pertinent rays in a plane containing the monopole; Fig. 4 shows the corresponding radiation patterns for each component. Thus, the problem is reduced to the superposition of the fields of three sources separated in space, and having different radiation patterns. [The patterns of the two edge sources are described in Section III and the monopole (or dipole) pattern is found in many references, e.g., [7].]

Figure 5 shows the measured and calculated patterns of the entire configuration for the case where the ground screen has a radius of three wavelengths and the monopole is a quarter wavelength long. It may be observed that there is good agreement between the measured and calculated patterns, especially in the periodicity of the interfering signals and also in the pattern amplitude near $\theta = 90°$. However, in the $\theta = 0$ and $\theta = 180°$ directions the calculated signal is significantly lower than the measured value. This is explainable by noting that in these two directions every point on the edge contributes a ray to the total field. These rays are all parallel and focus at infinity. This focusing action accounts for the increased signal level in these directions. In this application of the theory, the focusing was not taken into account. In many applications, it is sufficient to determine the patterns to the degree shown in Fig. 5. However, if required, techniques are available to take the focusing action into account [9].

## V. Impedance of a Monopole above a Finite Circular Ground Plane

Another application of the geometrical theory of diffraction in the realm of antenna theory is in the calculation of antenna impedance. As an example, impedance or the equivalent reflection is calculated for the same configuration discussed previously. For this particular problem, reflection concepts are employed since they are more compatible with the geometrical theory. The calculation of reflection is converted from three dimensions to the calculation of the reflection for a two-dimensional antenna. This problem is then treated as the special case of a two-dimensional radar antenna with the edges scattering energy back to the antenna.

Figure 6 shows the ray geometry in the near neighborhood of straight and circular edges excited, respectively, by line and point sources. The rays to the right are directly excited by the sources and those to the left are diffracted by the edges. The ratio of $d\beta$ to $d\theta$ represents the ratio of power density incident at the edge to the power density of the diffracted wave and is directly related to the two-dimensional radar cross section of the edges. According to the geometrical theory of diffraction, this ratio is approximately the same for the two cases being considered, provided that $k\rho' \gg 1$.

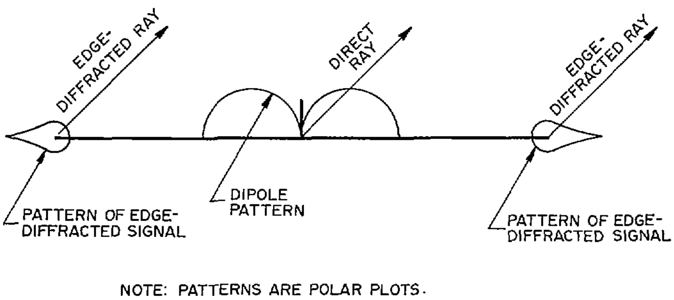



Fig. 4. Patterns and relative orientation and location of three sources contributing to the total field.

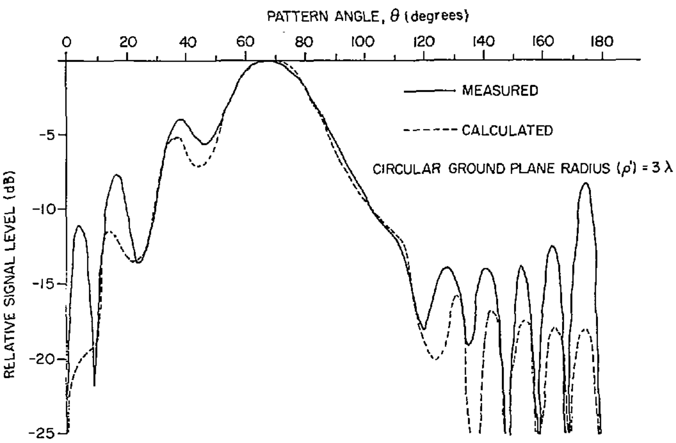



Fig. 5. Measured and calculated radiation patterns of a quarter-wave monopole operating over a finite circular ground plane.

The monopole is next considered to act as a transmitting antenna which excites a uniform radial wave in the azimuth plane that diverges in the elevation plane. The circular edge acts as a target which reflects a uniform radial wave in the azimuth plane that also diverges in the elevation plane as it converges on the monopole. Consequently, because of circular symmetry, only divergence in one plane has to be considered in determining the power ratio of the received to transmitted signals. From antenna theory, this ratio is given by

$$\frac{P_R}{P_T} = \frac{G}{2\pi\rho'} \frac{\lambda}{2\pi} \frac{1}{2\pi\rho'} \frac{G\lambda}{2}. \tag{7}$$

$G/2\pi\rho'$ = factor which converts transmitted power to power density at the edge (beam divergence in the elevation plane and antenna gain).

$\lambda/2\pi$ = two-dimensional radar cross section of the edges. (This factor is determined by the rigorous solution of the straightedge problem [8], with the condition that $k\rho' \gg 1$.)

$1/2\pi\rho'$ = factor which converts edge-scattered energy to power density at the antenna (divergence in elevation plane).

$G\lambda/2$ = effective height of the source antenna.

Combining terms, (7) becomes

$$\frac{P_R}{P_T} = \pi\left(\frac{G}{2\pi\rho'} \frac{\lambda}{2\pi}\right)^2.$$

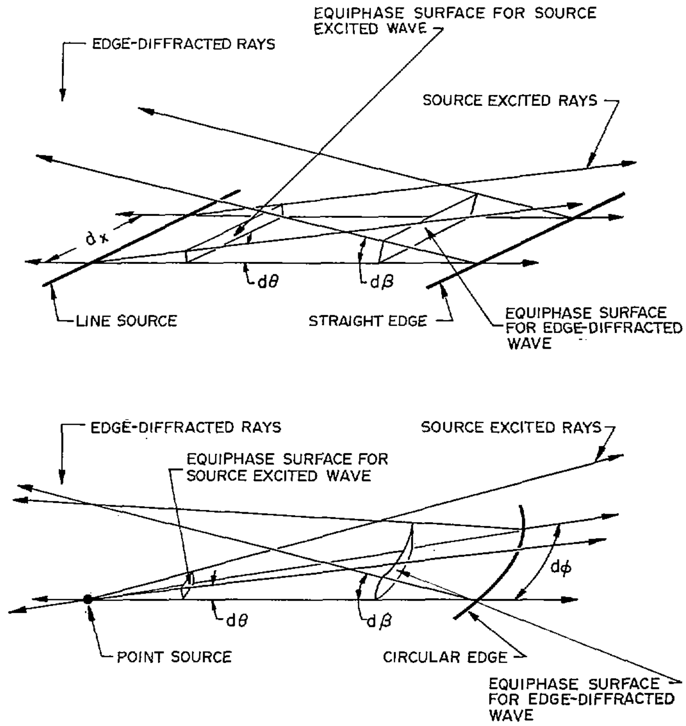



Fig. 6. Ray geometry for straight and circular edges in the near neighborhood of the edges.

The voltage ratio of received to transmitted signals is given by the square root of the power ratio and by the introduction of a phase factor.

$$\frac{V_R}{V_T} = \sqrt{\pi}\,\frac{G}{2\pi\rho'}\,\frac{\lambda}{2\pi}\exp j\left(\frac{\pi}{2} - 2k\rho'\right). \tag{8}$$

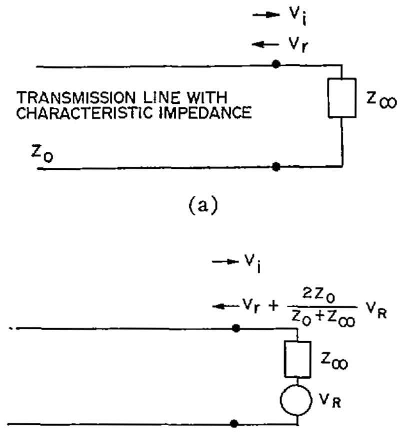
The constant phase factor, $\pi/2$ radians, is the residual phase of a $3\pi/4$-radian advance, introduced at the edges because of diffraction, and a delay of $\pi/4$ radians resulting from the focusing of the diffracted rays as they converge to the origin. (This latter phase factor is identical to the difference that exists between radial-waveguide reflection coefficients defined at the origin and at a large radius.)

The reflection of the disk-monopole antenna at its input port may now be calculated by adding the perturbation introduced by the edge of the ground plane to the reflection of a monopole operating in an infinite ground plane. For the case of the infinite ground plane, the equivalent network is shown in Fig. 7(a); it consists of a transmission line of characteristic impedance $Z_0$ feeding the antenna input port. The reflection and transmission coefficients at the antenna port for the infinite case are given by

$$\Gamma_\infty = \frac{V_r}{V_i} = \frac{Z_\infty - Z_0}{Z_\infty + Z_0} \tag{9}$$

$$\tau_\infty = \frac{V_T}{V_i} = \frac{2Z_\infty}{Z_\infty + Z_0}. \tag{10}$$

Figure 7(b) shows the equivalent network for the case of a finite circular ground plane. $V_R$ is an equivalent voltage source representing the contribution of the edge




(b)

Fig. 7. Networks for an antenna operating on infinite and finite ground planes.

reflection. The antenna reflection coefficient is now given by the sum of the infinite ground plane and edge-induced coefficients.

$$\Gamma = \Gamma_\infty + \Gamma_e. \tag{11}$$

$\Gamma_e$ is the portion of the edge reflected signal received by the antenna relative to the incident signal. It corresponds to the first reflection off the edge and, if the antenna-to-edge distance is large in terms of wavelengths, it is the dominant edge effect. If the distance is small, multiple edge reflections have to be taken into account, but these are neglected in this case. Considering the antenna as operating in the receive mode, the received signal from the edge relative to the incident is given by

$$\Gamma_e = \frac{2Z_0}{Z_\infty + Z_0} \frac{V_R}{V_i}. \tag{12}$$

From (8), solving for $V_R$ and dividing both sides by $V_i$,

$$\frac{V_R}{V_i} = \frac{V_T}{V_i} \sqrt{\pi} \frac{G}{2\pi\rho'} \frac{\lambda}{2\pi} \exp j\left(\frac{\pi}{2} - k\rho'\right). \tag{13}$$

If (10) is substituted for $V_T/V_i$,

$$\frac{V_R}{V_i} = \frac{2Z_\infty}{Z_\infty + Z_0} \sqrt{\pi} \frac{G}{2\pi\rho'} \frac{\lambda}{2\pi} \exp j\left(\frac{\pi}{2} - 2k\rho'\right). \tag{14}$$

Finally, substituting this result for $V_R/V_i$ in (12), the edge reflection coefficient is

$$\Gamma_e = \frac{2Z_0Z_\infty}{(Z_\infty + Z_0)^2} \frac{G}{\sqrt{\pi}k\rho'} \exp j\left(\frac{\pi}{2} - 2k\rho'\right). \tag{15}$$

Equation (15) may now be substituted in (11) for the desired result.

This result may be shown to be in good agreement with the results derived by J. E. Storer from the solution of an integral equation ([6], also summarized in [7]). Storer's result has been derived as an input impedance and is given by

$$Z = Z_\infty + Z_e$$

where $Z_\infty$ is the impedance of a monopole operating in an infinite ground plane and

$$Z_e = j\frac{60}{2k\rho'}\left|k\int_0^h \frac{I(x)}{I(0)}dx\right|^2 \exp(-j2k\rho').$$

The agreement between the results of this present paper and Storer's may be shown by converting the Storer result to the following reflection coefficient:

$$\Gamma = \Gamma_\infty + \frac{Z_0}{(Z_\infty + Z_0)^2}\frac{60}{k\rho'}\left|k\int_0^h \frac{I(x)}{I(o)}dx\right|^2 \cdot \exp j\left(\frac{\pi}{2} - 2k\rho'\right)$$

$$\Gamma = \Gamma_\infty + \Gamma_e'.$$

$\Gamma_e'$ is the edge induced coefficient according to Storer. Note the $\Gamma_\infty$ are identical for the two cases. For the case of a quarter-wave monopole,

$$\left|k\int_0^h \frac{I(x)}{I(o)}dx\right|^2 = 1$$

$$Z_\infty = 36.5\Omega$$

$$G = 1.64.$$

In order to make a comparison between the two results, the ratio of $\Gamma_e$, derived in this paper to $\Gamma_e'$, Storer's result, is determined to be

$$\frac{\Gamma_e}{\Gamma_e'} = \frac{2Z_\infty G}{\sqrt{\pi}60} = 1.13.$$

The ratio is 1.13 where perfect agreement would have been unity. Considering the difference in approach, this is reasonably good agreement. The phase is in exact agreement.

Here again, it should be emphasized that this particular example of a monopole above a circular ground plane was chosen for convenience and because a check was available in the literature. Actually, the concepts of two-dimensional antennas are not as widely employed as those for the typical three-dimensional antennas. In cases where ground planes with straightedges are employed, the geometrical theory of diffraction can be applied in conjunction with the usual notions of antenna gain and effective area. For example, for the case of a monopole centered on a square ground plane, the dominant edge-induced reflection component can be considered as caused by four point scatterers located in the plane of the ground plane, on lines perpendicular to the edges and passing through the monopole, and at twice the edge distance from the monopole. For this case, the effective area of the monopole is used in the calculation and no focusing of the diffracted rays needs to be considered.

## VI. Conclusion

The objective of this paper has been to demonstrate that the geometrical theory of diffraction provides a simple technique for calculating the radiation characteristics and impedance of complicated radiating structures. In essence, it resolves a difficult problem into component problems. The characteristics of the component problems are usually obtained from the literature or can be calculated and then combined to determine the characteristics of the initial problem.

For the radiation problem considered in this paper, the basic components were the diffraction by a straightedge and the radiation by a monopole in an infinite ground plane. A simple array calculation was used to combine the two components. For the impedance problem, the components were diffraction by a straightedge and the impedance of the monopole in an infinite ground plane. Radar principles were used to combine the two components to obtain the total impedance of the configuration considered.

## Acknowledgment

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# The Signal Produced in a Monopole Antenna by the Gamma Flux from a Nuclear Explosion

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***Abstract*—A monopole antenna is considered to be bombarded by a flux of gamma rays propagating normally to the axis of the antenna. As a result of this bombardment, electrons are scattered from the antenna, thus charging the antenna positively. This induces a current in the load impedance of the antenna—appearing as if the antenna were receiving a transient RF signal. An expression for the time dependent voltage across the load impedance of the antenna is obtained by considering that every electron scattered from the antenna never returns and that the antenna impedance and capacitance are their free-space values.**

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R. H. Duncan is at the New Mexico State University, University Park, N. Mex.

## Introduction

According to experimental observations, a transient signal is induced in a receiving antenna, or probe, by the high-density gamma flux emitted from a nuclear explosion. When an object is irradiated by this flux, it becomes positively charged because of electrons being scattered from it by virtue of the Compton effect. An attempt is made in this paper to explain the transient signal produced in an irradiated antenna by considering the temporal charging of the antenna to be exciting the receiving circuit.

An exact treatment of the interaction of gamma rays with an antenna is not feasible. The electromagnetic field generated by the interaction is composed of two parts: that attributable to the antenna and that generated by the space current of scattered electrons. How-