WIDEBAND IMPEDANCE MATCHING

by

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I. Introduction

This is the topic scheduled for a staff meeting 500518, the eighth and last of this season. It is the fourth talk on "Introduction to Advanced Theory of Communication Networks".

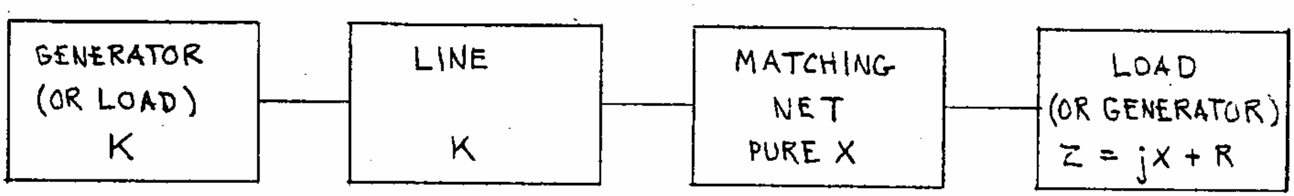
II. Matching with a Reactive Load

Present treatment is limited to:

1. Generator having resistance determining the power available therefrom.
2. Load having resistance which is to receive as much as possible of the available power.
3. Integrating type of reactance in load <u>or</u> generator limiting the frequency bandwidth of efficient power transfer.
4. Matching network including only pure reactance coupling the generator with the load.

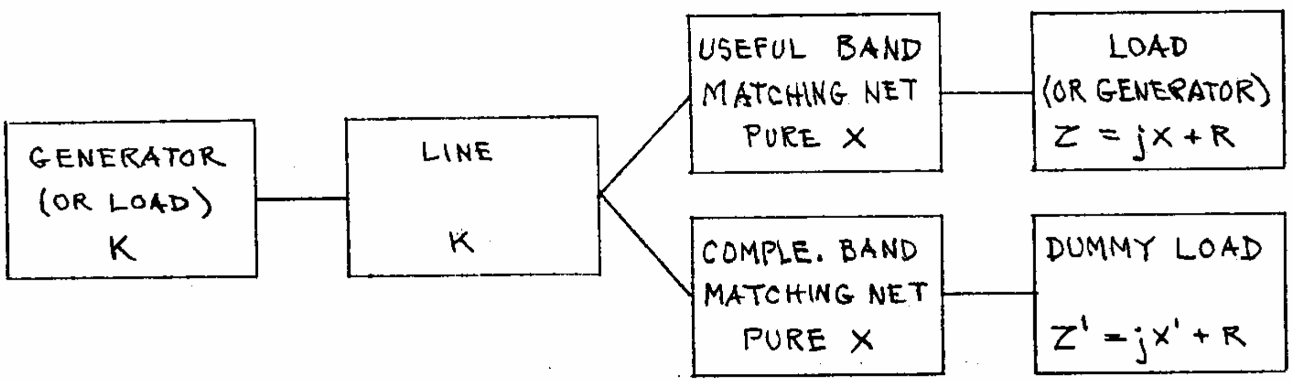
The simple presentation of this subject is neglected in the literature. This treatment introduces some of the simplest problems and their most general solutions.

Fig. 1. - System including reactance network between generator and load.



The generator (or load) has pure constant resistance K.
The line, which is optional, has wave resistance K; since it is adjacent to one matched termination K, it retains the same efficiency of matching and does not complicate the problem.
The load (or generator) has mixed impedance, Z = jX+R.
The matching net comprises pure reactors designed to reduce the mismatching that would be caused by the reactance in one termination Z.

Fig. 2 - System including complementary-band dummy load.



In some cases, the K termination must be matched very closely, even at the expense of efficiency; this system is the most efficient way of accomplishing this result.
The two matching networks are complimentary filters which cooperate to present a constant resistance K, ideally at all frequencies, and practically over a range much wider than the useful band.
This system is used in television transmitters to minimize line reflections (echoes).
Since maximum efficiency is usually the primary objective, this system receives no further attention herein.

Detrimental effects of mismatching.

If two terminations, real K and complex $\underline{Z}$ are joined, there is a voltage-reflection ratio, in complex form:

$$\underline{w} = \frac{\underline{Z}/K-1}{\underline{Z}/K+1}$$

If the impedance ratio $\underline{Z}/K$ is plotted as a point on the hemisphere chart, the radius to this point is the reflection ratio $\underline{w}$.
WM-4; also R-409, "Generators in networks", p. 7.
This causes a reflected signal in the line in Figs. 1 and 2, with corresponding reflection loss in transmission, so the load receives less than the available power from the generator, in the ratio $1-w^2$. (In this case, one termination being pure resistance, "reflection" and "transition" loss are the same.)
If the line is included, and the mismatch is at the load end (not the generator end), the reflection causes a standing wave in the line, whose voltage or current ratio is expressed as follows in terms of the magnitude w of the reflection ratio of voltage or current:

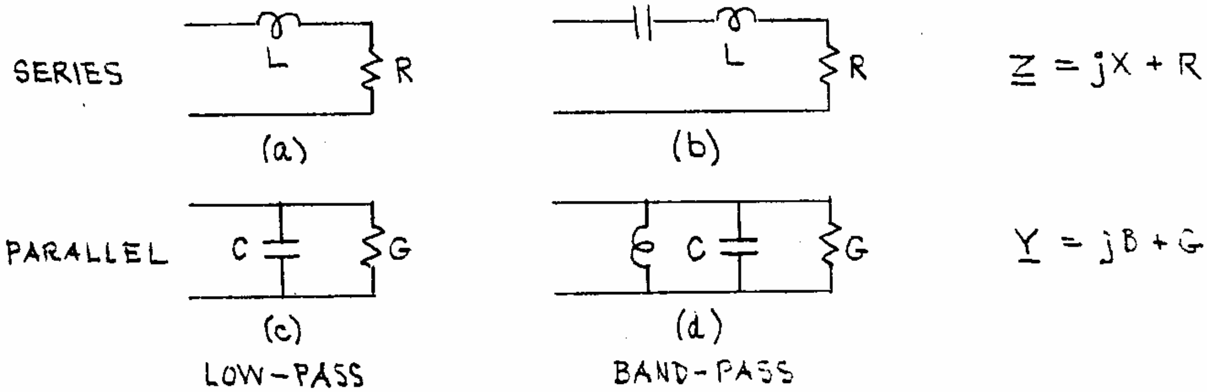
$$S = \frac{1+w}{1-w} > 1$$

To the extent that S exceeds unity, it increases the power loss and the peak voltage in the line.
The increase of power loss reduces the transmission efficiency and the carrying capacity of average power with limited heating; in a long line of small attenuation the power loss and heating are multiplied by

$$\frac{1+w^2}{1-w^2} = \frac{1+S^2}{2S}$$

If there is a minor ratio reflection at only one junction, the reflection loss is small, but multiple reflection at several junctions may cause a disproportionate amount of loss.
Hazeltine Report 1414W, 1942, 2-color hemisphere chart with radial scales; also P. H. Smith, Electronics, p. 29, Jan. 1939; also circular circulator described in this Smith paper.

Fig. 3 - Simple reactive loads.



These simple series and parallel types of reactive loads form the basis for the further discussion herein.

The series type will be presumed, since the parallel type is merely the invert (dual) thereof.

The impedance angle ϕ of impedance is common to both:

$$\tan \phi = X/R \text{ or } B/G$$

In the low-pass type, the frequency is measured by the ratio,

$$s = \tan \phi = \omega L/R \text{ or } \omega C/G$$

In the band-pass type, the relative departure from mid-frequency is measured by a similar ratio:

$$s = \tan \phi \approx 2\Delta\omega L/R \text{ or } 2\Delta\omega C/G = 2Q\Delta\omega/\omega_o = 2\Delta\omega/p\omega_o$$

The approximation is close for high values of $Q = \omega_o L/R$ or $\omega_o C/G$, corresponding to small values of the dissipation (power) factor $p = 1/Q$.

The departure from mid-frequency is $\Delta\omega$.

Bandwidth

The above low-pass and band-pass types of reactive impedance have their bandwidth limited by the associated reactors of "integrating" type, namely, series L or shunt C; the other reactor in the band-pass type merely tunes to resonance at mid-band.

The bandwidth for all types is $\omega_w = 2\pi f_w$.

If there is a nominal bandwidth of operation, the properties of the reactive load at the edges of the band (other than zero frequency) are s_c, ϕ_c, etc.; the corresponding reflection factor w_c is presumed also to be the maximum occurring in any part of the pass band.

Fig. 4 - Typical antennas as tuned reactive loads.

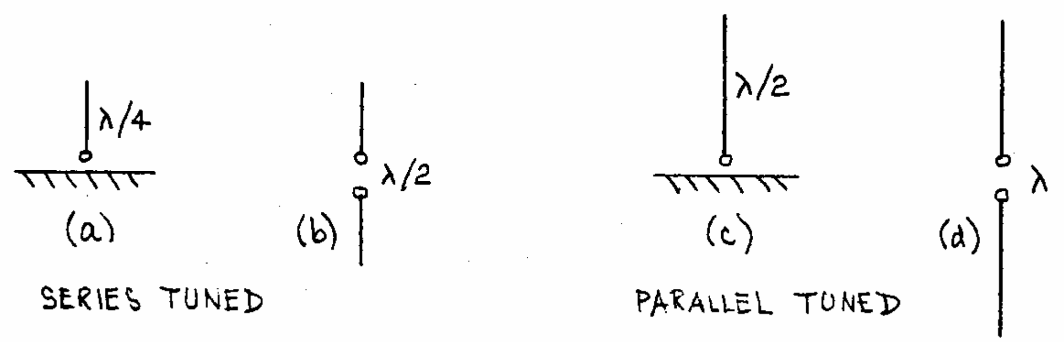
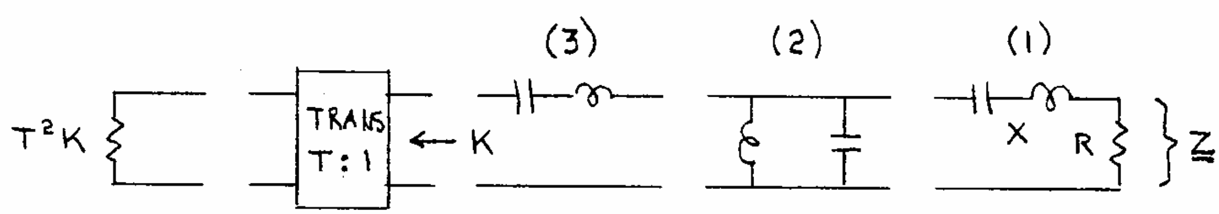


Fig. 5 - Simple technique of wideband matching.



The load Z includes a single tuning (1); each successive added reactance arm includes another tuning (2), (3), etc.; a transformer may be included to adjust the impedance ratio.

Filter technique

Wheeler & Whitman, "The design of doublet antenna systems", Proc. I.R.E., vol. 24, pp. 1257-1275; Oct. 1936.

In the filter technique, the inherent reactance in the load is taken to be the mid-series or mid-shunt arm of a constant-K filter; hence this arm determines the nominal band-width of the filter:

$$R = \omega_w L \quad \text{or} \quad G = \omega_w C \; ; \quad \omega_w = R/L \quad \text{or} \quad G/C$$

$$s_c = 1 \; ; \quad \phi_c = \pi/4$$

The bandwidth is inversely proportional to the time constant L/R or C/G. If K = R, the matching is excellent near mid-band; however, the matching over the entire band (or any fraction thereof) may be held within closer limits by some degree of mismatching at midband, as will be explained.

Best matching

If matching is equally important over the entire width of a specified band, it is customary to specify as a tolerance the maximum w or S that occurs within the band, denoted w_c or S_c because this maximum value always occurs at cutoff in an optimum design.

The technique of best matching will be described further on; a few of the rules are as follows:

Since the series L or shunt C directly associated with R is the limiting factor, it must not be supplemented in the matching network; hence the added second reactance arm is oppositely connected (parallel or series).

The best matching can be secured by a ladder of alternating parallel and series arms; they are so designed that every peak of reflection factor in the band is equal to its cutoff value w_c; also so that the intervening valleys are as high as possible, for reasons that will be mentioned.

In general, adding more reactance arms can improve the matching within any specified band, at the expense of worse matching outside the band; however, the limit of performance is approached rapidly for added arms beyond the second.

If certain tolerances are required on the reactance arms, there is an optimum number of arms, beyond which the harm from the tolerances exceeds the good from the added arms; any limitation on the labor of design imposes one "tolerance" in this respect.

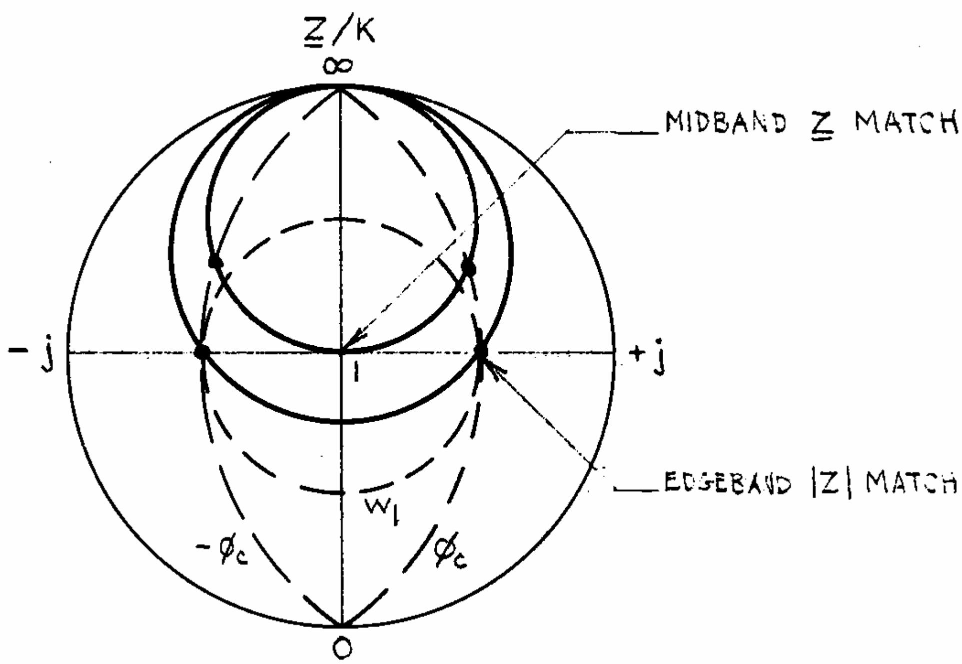
III. Single-Tuned Matching

If the objective is wideband matching, the only tuning is that associated with the reactive load, adding no inductance directly in series and no capacitance directly in parallel.

The single tuning is denoted (1) in Fig. 5, and the transformer is included if needed.

WM-4, Circle diagrams (w = reflection ratio)

Fig. 6 - Single tuning plotted on hemisphere chart.



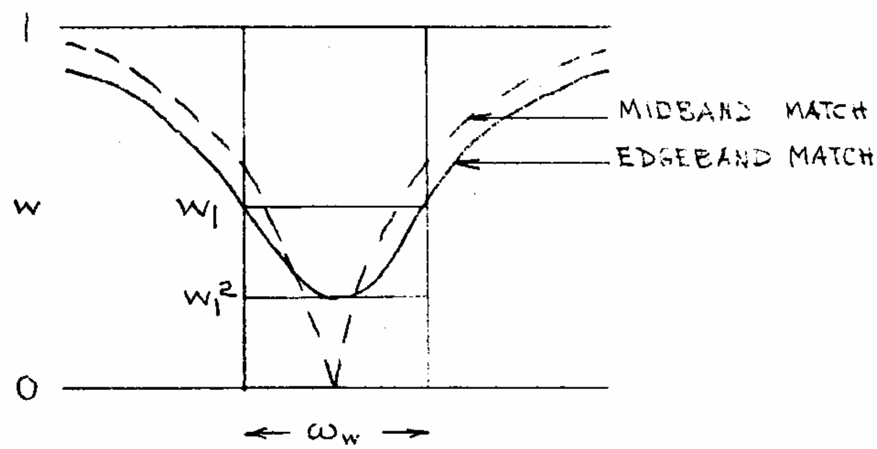
A simple series-tuned impedance (with constant resistance) has a locus which is a circle tangent to the rim at the upper pole.
At the edges of the frequency band, this circle intersects the meridians of $\pm \phi_c$.
The smallest radius within which the intervening arc can be contained is

$$w_1 = \tan\phi_c/2$$

This arc is matched in magnitude of impedance at edgeband, but mismatched (to a radius w_1^2) at midband.
The arc is also plotted matched at midband, to show the slightly greater radius to the edgeband points:

$$w_1' = \frac{\tan\phi_c}{\sqrt{4+\tan^2\phi_c}} = \frac{1+\sqrt{1+\tan^2\phi_c}}{\sqrt{4+\tan^2\phi_c}} \tan\phi/2 > \tan\phi_c/2$$

Fig. 7 - Reflection ratio of single tuning.

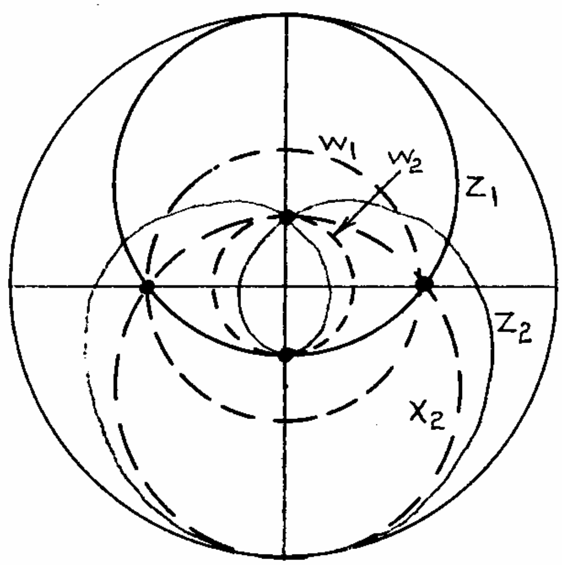


The amount of mismatching is more nearly uniform and falls within the closest limit if matched edgeband rather than midband.

IV. Double Tuned Matching

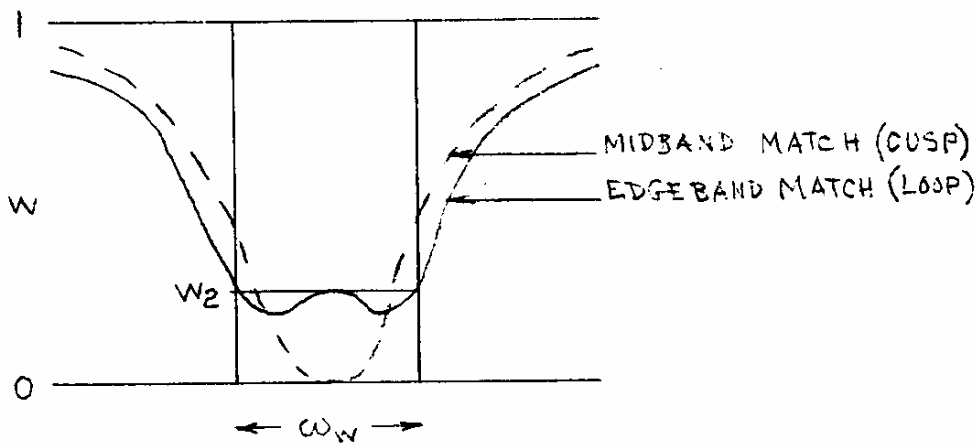
Double tuning is obtained by one added tuning such as (2) in Fig. 5. In this case the added tuning is in parallel since the load tuning is in series.

Fig. 8 - Double tuning plotted on hemisphere chart.



The added tuning has the property of shifting the edgeband points of Z_1 (single tuning) around the circle Z_2; it can be adjusted to loop the curve within the radius w_2 which is the resulting reflection ratio at edgeband and midband.
Note the simple relation, $w_2 = w_1^2 = \tan^2 \phi_c/2$, which is the improvement possible by double tuning.
This relation has a simple geometric proof on the hemisphere chart whereas its algebraic proof is laborious.

Fig. 9 - Reflection ratio of double tuning.

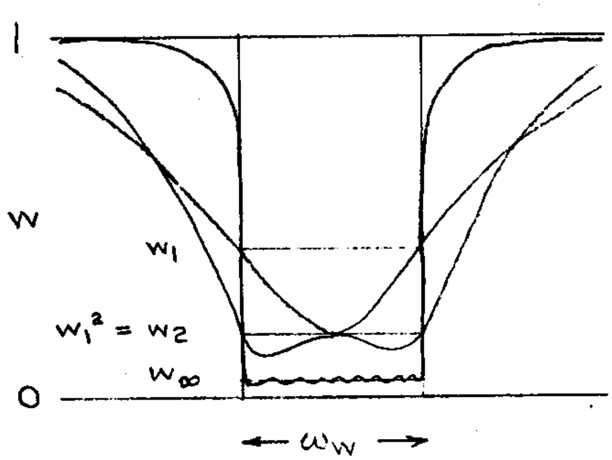


The dotted curve is plotted for an adjustment of double tuning which gives a cusp (not a loop) at the center of the hemisphere chart; it is closer near midband but goes out to a greater radius at edgeband.

V. Multi-Tuned Matching

Bode, "Network Analysis", Sec. 16.3.
Fano, "Theoretical limitations on the broadband matching of arbitrary impedances", MIT Research Lab. of Electronics, Report 41, Jan. 1948. (HAW has copy.)

Fig. 10 - Reflection ratio of multituning.



The addition of many tuned circuits, adjusted for holding the reflection coefficient within the closest limit, causes the curve to approach a lowest uniform value, w_∞ determined by ϕ_c at the edges of the required bandwidth ω_w.

On the hemisphere chart, n-1 added tuned circuits causes n-1 loops around the center just within the radius w_∞.

The formula which limits the closeness of matching over any bandwidth is

$$\int_0^\infty (\log 1/w)\,d\omega < \pi R/L \quad \text{or} \quad \pi G/C$$

Just as w is a measure of the degree of mismatching, log 1/w is a measure of the matching, being zero at the rim of the hemisphere chart and increasing toward the center.

On this basis, the above integral expresses the "area of matching" as being inversely proportional to the series L or shunt C of the load. Ideal matching over the bandwidth ω_w by adding many tuned circuits enables the reflection ratio to be reduced to a nearly uniform value determined by the above integral:

$$\log 1/w_\infty = \frac{\pi}{\tan\phi_c} \;;\quad w_\infty = \exp - \frac{\pi}{\tan\phi_c}$$

The bandwidth ω_w is proportional to $\tan\phi_c$ in a low-pass net and approximately so in a band-pass net.

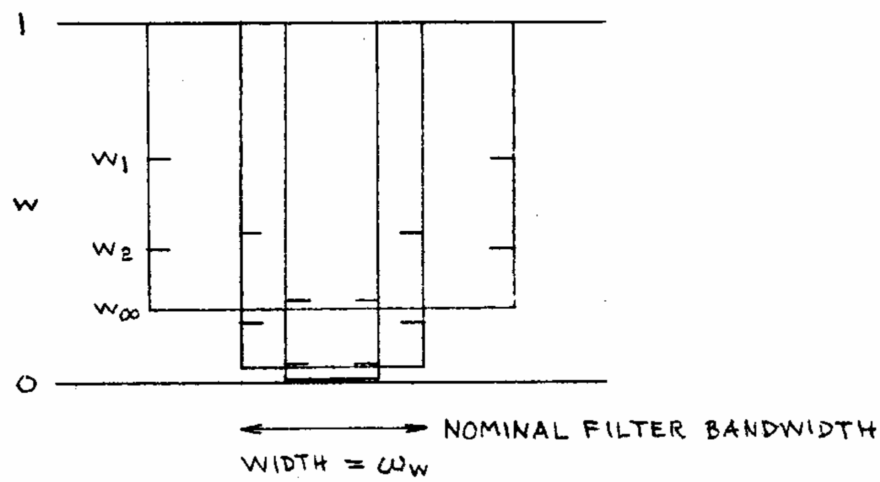
The above curves are plotted for the nominal filter bandwidth, for which $\phi_c = \pi/4$.

Single tuning:	$w_1 = \tan \pi/8$	$= 0.415$
Double tuning:	$w_2 = \tan^2 \pi/8$	$= 0.172$
Manifold tuning:	$w_\infty = \exp -\pi$	$= 0.043$

This limiting value is approached very slowly with increasing number of tunings, so only double or triple tuning is likely to be justified in practice.

With practical tolerances on the added reactance arms, there is an optimum number of added tunings, such that a greater number would increase the reflection ratio by their tolerances more than they would decrease it by the theory.

Fig. 11 - The variation of reflection ratio with bandwidth.



The value of reflection ratio varies as follows with bandwidth:

Bandwidth	Reflection ratio		
Filter bandwidth	n = 1	2	∞
$\tan\phi_c$ = 2	.618	.381	.208
1	.415	.172	.043
1/2	.236	.056	.002

Closer matching is obtained by uniform reflection ratio in the band and by sharper cutoff, so as not to "waste" any matching inside or outside of the band.

Degrees of freedom. If the load is resonant, only one more parameter (the transformer ratio) is needed to match at midband or edgeband. Every loop around the center requires two more parameters to close the loop, and these can be provided by one added tuned circuit.

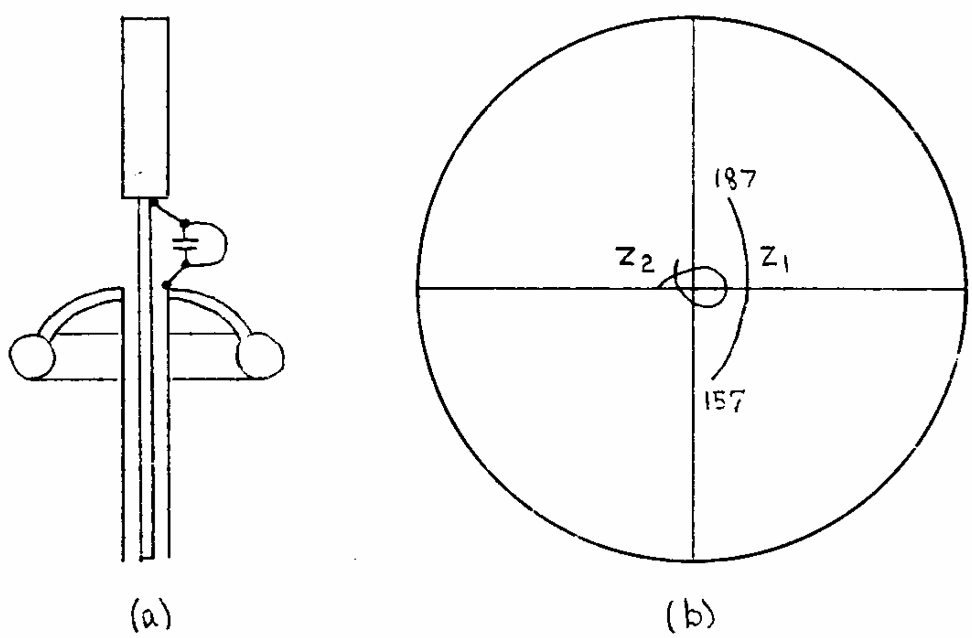
VI. Antennas

Typical examples of multituning for impedance matching are associated with resonant antennas such as those in Fig. 4.

The above reference (Wheeler-Whitman) introduces the filter method; this method is the only one simple to compute; the first 2 or 3 added tunings are well utilized, while there is little or no further benefit by adding more.

WM-1, Fig. 19, shows several double-tuned antennas adapted for wideband matching.

Fig. 12 - The "life-saver" antenna and its impedance plot.



This antenna was the first application of optimum double-tuned matching in HEC antennas for World War II (designed by antenna group including DD and HAW).
HAW, "BL lifesaver antenna", HEC Report 1507, May 1943.
Antenna (a) is quarter-wave radiator, providing Z_1 single-tuned and series-resonant; added tuned circuit is parallel-resonant, providing Z_2 double-tuned.

Frequency range	157-187 Mc
Frequency midband	$f_o = 172$ Mc
Frequency bandwidth	$f_w = 30$ Mc
Z_1 in circle of radius	$w_1 = \tan\phi_c/2 = 0.35$
Angle at edgeband	$\phi_c = 39^\circ$
	$\tan\phi_c = 0.81$
Resonance ratio	$Q = (f_o/f_w)\tan\phi_c = 4.6$
Z_2 in circle of radius	$w_2 = w_1^2 = 0.12$

The last value is based on the theory and agrees with the experimental adjustment of this design.

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NB 43, pp. 114-126.
J-102, J-211.