Sharp Cutoff Radiation Patterns

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Abstract—**A method is described for estimating the available slope (dB/deg) for the design of a sharp cutoff radiation pattern with a given aperture size, sidelobe level, and ripple factor. Curves are presented which relate slope factor (dB/deg/wavelength of aperture), sidelobe level, and ripple factor. These curves are obtained by defining and evaluating a Chebyshev integral pattern function which is representative of a sharp cutoff radiation pattern.**

I. INTRODUCTION

A CLASSIC PROBLEM in antenna pattern synthesis is the design of a sector pattern (see Fig. 1) for a linear or planar array with the sharpest cutoff for a given sidelobe level, ripple factor, and aperture size. This problem is receiving more attention recently because of the increased use of array antennas to reduce the so-called "elevation lobing" effect typical of ground-based antennas operating at microwave frequencies [1]–[3]. More recently, the development of the microwave landing system (MLS) has indicated the need for a sharp cutoff and low sidelobe design for the vertical-plane pattern of the azimuth antenna to suppress guidance errors caused by reflections from a possible uneven or tilted ground, in front of the antenna.

Some historical background is helpful in defining and understanding the various aspects of the problem. In 1952 Ruze [7] proposed that the convolution of the Chebyshev pattern with a unit step pattern should result in a pattern with near-maximum slope for a given deviation and aperture size. His method falls into a category which is now called a linear-phase-pattern synthesis. The resulting pattern approaches an equal sidelobe and equal ripple pattern; the sidelobe level and ripple factor are not independent of each other. Later, Schell and Ishimaru [8] indicated that a closer approximation to a step pattern can be achieved with a nonlinear-phase-pattern (power pattern or complex) synthesis method. Steyskal [9] utilized Chebyshev sampling functions and a power-pattern synthesis to approach the equal sidelobe and equal ripple pattern. Recently, Evans [3] has described a synthesis method which is believed to provide the optimum result. He applied the results of modern digital filter synthesis theory [10] to the antenna-pattern synthesis problem and indicates 1) that for a linear-phase synthesis of a sharp cutoff pattern, the optimum pattern has equal sidelobes and equal ripples (equiripple pattern) and that the sidelobe level and ripple factor can be specified independently of each other; and 2) that a minimum-phase-pattern (complex) synthesis provides the closest approximation to the desired step pattern. The minimum-phase-pattern synthesis also allows for independent specification of sidelobes and ripples; it must also be an equiripple pattern. Evans [3], based on the work of Rabiner *et al.* [10], suggests that relative to a linear-phase synthesis a 10- to 30-percent reduction in aperture size can be achieved with a minimum-phase synthesis for a desired slope factor, sidelobe level, and ripple factor. It should be noted that the optimum minimum-phase result can also be approached by means of the elliptic function method which is widely used in filter design [14].

From the above discussion it is clear that optimum synthesis techniques for a sharp cutoff radiation pattern are available in the literature; however, the basic trade-off among the design parameters is somewhat difficult to interpret and evaluate. The objective of this paper is to provide a simple method for a system designer to estimate the slope that can be obtained as a function of aperture size, sidelobe level, and ripple factor. This is accomplished by defining and evaluating a Chebyshev integral pattern which is representative of a sharp cutoff pattern. The key building block in the development of this pattern function is the Chebyshev Fourier transform pair [4]–[6].

II. THE CHEBYSHEV INTEGRAL PATTERN FUNCTION

The ideal (infinite slope, zero sidelobes, and zero ripples) pattern function is the unit step function. Fig. 1 shows the relevant aperture geometry and Fig. 2 shows the Fourier transform pair (aperture excitation and radiation pattern) for the unit step pattern function. A well-known pattern synthesis technique for approximating a desired pattern shape with a finite-aperture size is to convolve the desired pattern with a narrow-beam low sidelobe pattern of a finite aperture [7], [8]. The convolution process in the pattern domain corresponds to a multiplication process in the excitation domain.

To achieve sharp cutoff, low sidelobes, and small ripple by convolving the unit step with a finite-aperture function, the latter should have a narrow beam and low sidelobes; thus the Chebyshev pattern function $CB(u)$ (smallest beamwidth for a given sidelobe level) should be a near-optimum choice for the finite-aperture function. Fig. 3 shows the Chebyshev Fourier transform pair [6].

The convolution of the unit step and Chebyshev pattern functions results in a Chebyshev integral pattern function $CBi(u)$. The Fourier transform pair is shown in Fig. 4. Fig. 5 shows a typical Chebyshev integral radiation pattern. It is noted that the pattern is not quite equiripple (equal sidelobes and equal ripples). The Chebyshev integral can be modified by decreasing the area of the impulses at the aperture edges so that the pattern is near equiripple. The slight improvement in slope factor, however, is small (see Fig. 7). Fig. 6 shows an equiripple pattern. This pattern was obtained by decreasing the impulses at the edges of the aperture by 6.8 dB; the original Chebyshev integral pattern had −15.9-dB sidelobes.

III. DEFINITION OF SLOPE FACTOR

The choice of the most useful point on the skirt of the cutoff pattern to define the slope factor depends on the appli-

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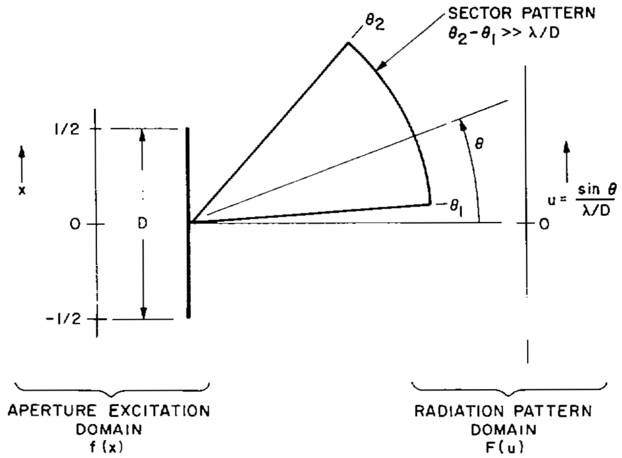

Fig. 1. Geometry.

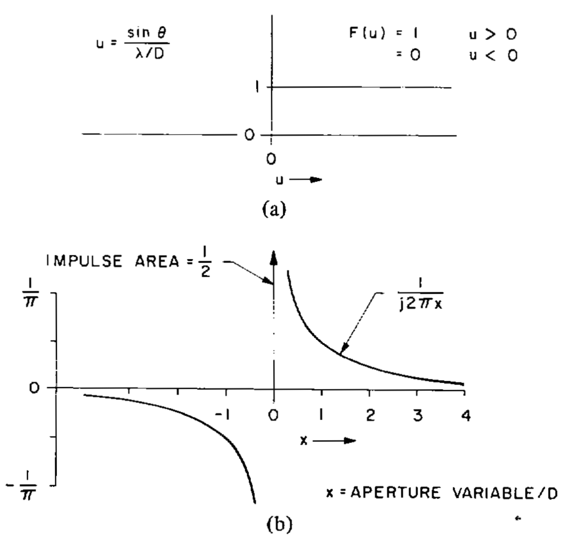

Fig. 2. Unit step Fourier transform pair. (a) Pattern function. (b) Excitation function.

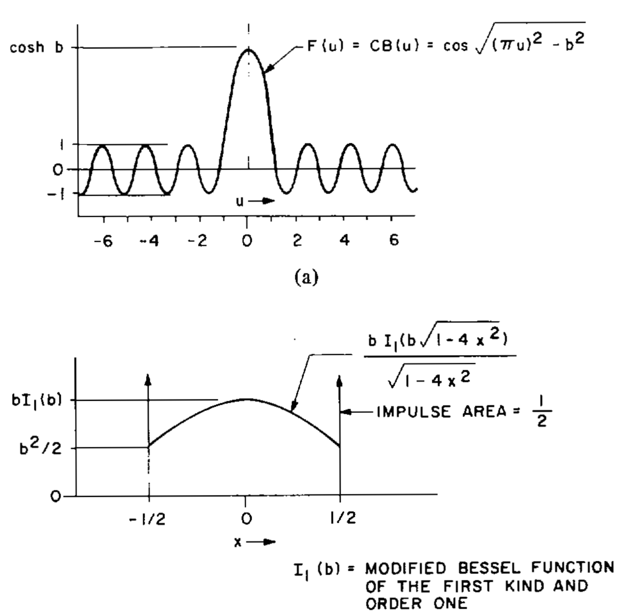

Fig. 3. Chebyshev Fourier transform pair. (a) Pattern function. (b) Excitation function.

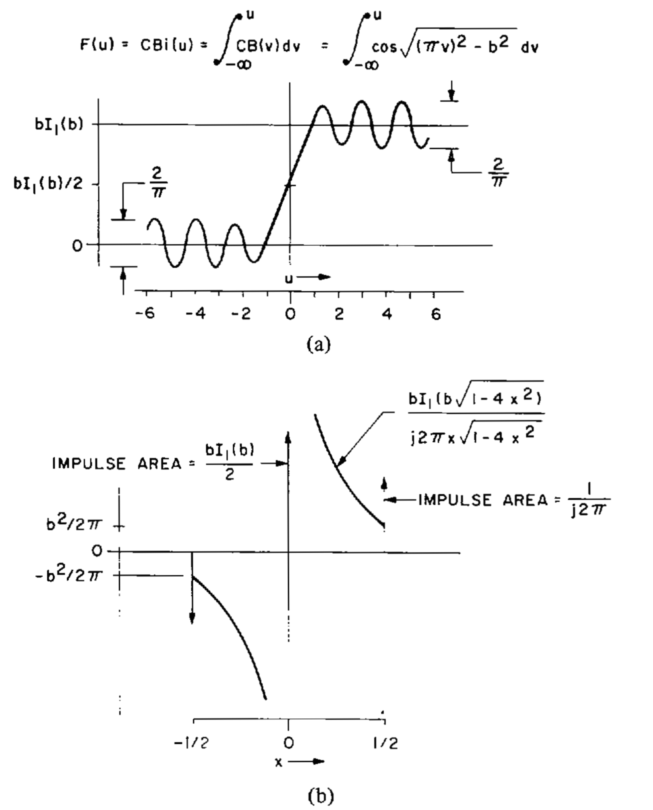

Fig. 4. Chebyshev integral Fourier transform pair. (a) Pattern function. (b) Excitation function.

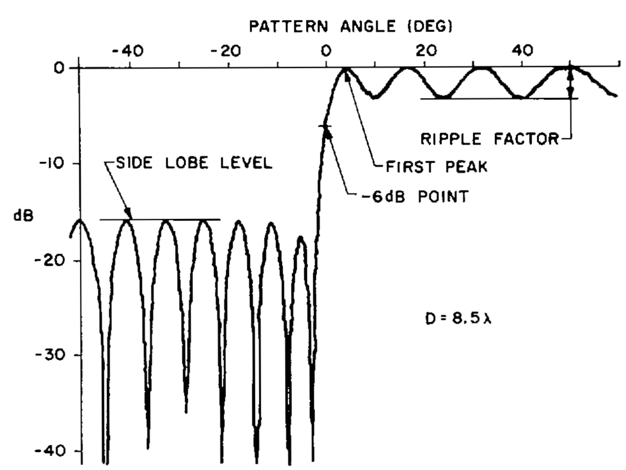

Fig. 5. Typical Chebyshev integral pattern.

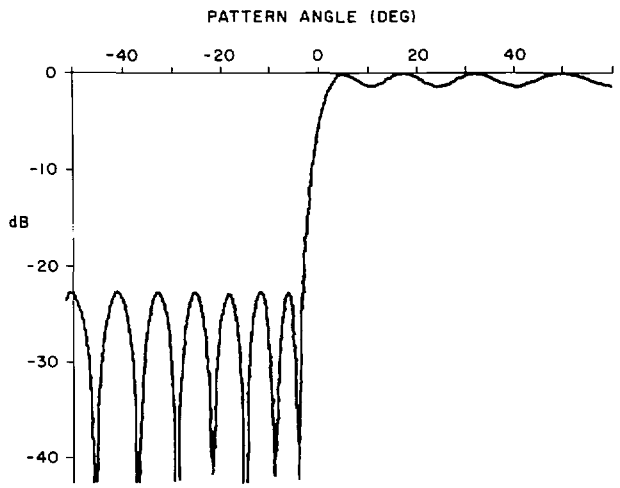

Fig. 6. Modified Chebyshev integral pattern for constant sidelobes and ripples.

cation. In the past the following points have been used: the points corresponding to 3, 6, and 12 dB down from the first peak and also the maximum slope point (in voltage ratio per unit angle). The maximum slope point can vary from 6 dB down from the first peak (for very low sidelobe patterns) to more than 20 dB down from the first peak (for very high sidelobe patterns).

Experience with the design and application of sharp cutoff patterns indicates that the −6-dB point (6 dB down from the first peak) is perhaps the best overall choice. In this paper the slope factor (dB/deg/wavelength of aperture) is defined at the −6-dB point:

$$\text{slope (dB/deg)} = \frac{\pi}{180}\,\frac{d}{d\theta}\,dB(F(u)) = \frac{\pi}{9 \ln 10}\,\frac{\cos\theta}{\lambda/D}\,\frac{\frac{d}{du}F(u)}{F(u)},$$

$F(u)$	pattern function,
u	$(\sin\theta)/(\lambda/D)$,
D/λ	aperture size in wavelengths,
θ	angle from line perpendicular to aperture (see Fig. 1),
dB(·)	20 log (·).

The slope factor (SF) = slope/(D/λ); at the −6-dB point it is given by

$$SF(\text{at} - 6\text{ jB}) = \frac{\pi}{9\ln 10}\cos\theta_{-6\,\text{dB}}\left.\frac{\frac{d}{du}F(u)}{F(u)}\right|_{u=u_{-6\,\text{dB}}} \cdot (\text{dB}/^\circ/(D/\lambda)).$$

The slope factor as defined above is helpful because the slope, in dB/deg, is easily determined by multiplying the slope factor by the aperture size in wavelengths.

IV. RELATIONSHIP OF SLOPE FACTOR, SIDELOBE LEVEL, AND RIPPLE FACTOR

In this section the relationship of slope factor, sidelobe level, and ripple factor is determined for the Chebyshev integral pattern functions. (It is assumed that the −6-dB point is broadside to the array aperture, that is, $\theta_{-6\text{ dB}} = 0$.) Two limiting cases are easily derived; these are the 0-dB sidelobe case and the case for low sidelobes.

The Chebyshev integral pattern function for 0-dB sidelobes and for the −6-dB level at $\theta = 0^\circ$ is simply

$$F(u) = \text{CBi}\,(u) = \frac{\sin \pi(u - 1/6)}{\pi}$$

$$\left.\frac{\frac{d}{du}F(u)}{F(u)}\right|_{u=0} = \pi\sqrt{3}$$

$$\text{SF} = 0.82\ (dB/^\circ/(D/\lambda)).$$

In the limit of low sidelobes the slope at the −6-dB point approaches the maximum slope case (V/unit angle) and is determined by evaluating the Chebyshev integral and its derivative at $u = 0$:

$$F(0) = \text{CBi}\,(0) = \int_{-\infty}^{0} \text{CB}\,(u)\,du = \tfrac{1}{2}\,bI_1(b)$$

$I_1(b)$ = modified Bessel function of the first kind and order one

$$\left.\frac{\frac{d}{du}F(u)}{F(u)}\right|_{u=0} = \frac{\cosh b}{bI_1(b)/2}.$$

The slope factor is given by

$$\text{SF} = \frac{2\pi}{9\ln 10}\,\frac{\cosh b}{bI_1(b)}.$$

The sidelobe level (voltage ratio) (SL) and ripple factor (maximum voltage/minimum voltage) (RF) are determined by inspection from Fig. 4:

$$\text{SL} = \frac{1}{\pi b I_1(b) + 1}$$

$$\text{RF} = \frac{\pi b I_1(b) + 1}{\pi b I_1(b) - 1}.$$

The above parametric relationship of SF, SL, and RF provides a good approximation (less than three-percent error) for sidelobes below −26 dB. The slope factor and ripple factor for intermediate sidelobe levels were evaluated by means of digital computation; the results are presented in Fig. 7. The relationship shown in this figure is believed to be near optimum for a linear-phase synthesis with respect to the maximum slope factor for the specific set of sidelobe levels and ripple factors.

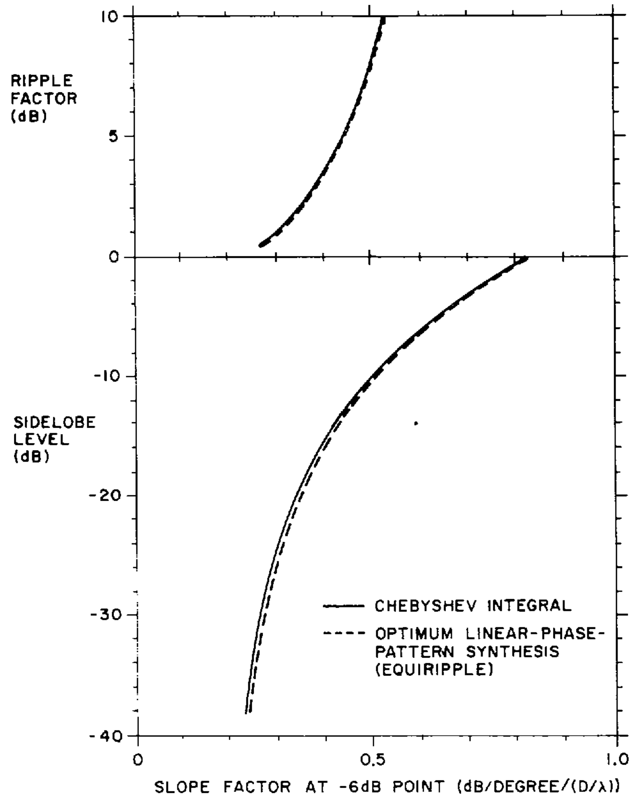

Fig. 7. Relationship of slope factor, sidelobe level, and ripple factor.

If the sidelobe and ripple requirements are different from the parametric relationship of Fig. 7, an averaging (interpolation) of the slope factors for the desired sidelobe and the desired ripple should provide a close approximation of the slope factor for the desired sidelobe and ripple combination provided that the difference in slope factors is small. For example, if a −24-dB sidelobe level and a 3-dB ripple factor design is desired, the maximum slope factor is estimated by averaging the slope factors for the −24-dB sidelobe level (0.30) and the 3-dB ripple factor (0.38); the result is a slope factor of 0.34 dB/deg/(D/λ). Experience has shown that the above procedure provides good approximations if the two slope factors do not differ by more than ±0.1. As noted in the Introduction, if a minimum-phase (complex) synthesis is used, it is expected that in some case a 10- to 30-percent increase in the slope factor can be achieved [3], [10].

Although the above results were derived for a unit step pattern, they are also believed to be representative of the cutoff characteristics for a sector pattern provided that the sector width is several times greater than λ/D. This is based on computations of sector patterns where the sector width was varied to determine if there is any special effect attributed to the interaction of the two edges of the sector pattern and also on a comparison with computed patterns for some actual antennas.

Table I presents a comparison of the computed slope factors for some actual array antennas having sharp cutoff patterns and the values estimated by the averaging method described above. A comparison of the last two columns indicates that the estimated and computed values are in good agreement except for the first antenna, where other design characteristics took preference over the maximization of the slope. For the

TABLE I
COMPARISON OF THE SLOPE FACTOR COMPUTED FOR SOME ACTUAL ANTENNAS AND THE SLOPE FACTOR ESTIMATED BY THE METHOD DESCRIBED IN THIS PAPER

ANTENNA	NOMINAL SIZE	DEVELOPER (REF.)	COMPUTED VERTICAL APERTURE DESIGN: SYNTHESIS METHOD	APERTURE SIZE** D/λ	SIDELOBE LEVEL DB	RIPPLE FACTOR DB	SLOPE FACTOR DB/°/(D/λ)	ESTIMATED SLOPE FACTOR* DB/°/(D/λ)
E-SCAN CIRCULAR ARRAY	8'x40'	HAZELTINE [1] [2]	POWER PATTERN	8.5	-23	1.6	.28	.32
DABSEF PLANAR ARRAY	8'x22'	HAZELTINE [13]	POWER PATTERN	8.4	-24	1	.35	.30
4' OPEN ARRAY PLANAR ARRAY	4'x25'	HAZELTINE [11]	LINEAR PHASE	4.0	-24	2	.31	.32
5' OPEN ARRAY PLANAR ARRAY	5'x25'	HAZELTINE [12]	LINEAR PHASE	5.0	-11	4	.45	.44
PALM LINEAR ARRAY	4'	LINCOLN LAB/ WESTINGHOUSE [5]	POWER PATTERN	4.5	-13.5	3.5	.41	.40
ATCRBS PLANAR ARRAY	8'x27'	WESTINGHOUSE [15]	LINEAR PHASE	8.6	-22	2.5	.32	.34

*OBTAINED BY USE OF THE CHEBYSHEV INTEGRAL CURVES OF FIG. 7 (NEAR OPTIMUM LINEAR-PHASE SYNTHESIS)
**APERTURE SIZE = NUMBER OF ELEMENTS × ELEMENT SPACING/WAVELENGTH

case of the DABSEF antenna the higher computed value (17 percent) is attributed to the power-pattern synthesis, which is known to provide a higher slope than the linear-phase synthesis.

In Table I a comparison of the 5-ft open array and the PALM array leads to an interesting conjecture. The former used a linear-phase synthesis while the latter used the optimum minimum-phase synthesis. A comparison of the slope factors shows no substantial increase of the minimum-phase value as compared to the linear-phase value. Both of these cases have high sidelobe and high ripple factors. This leads to the belief that the results of the minimum-phase and linear-phase syntheses approach each other in the limit of high sidelobe levels and ripple factors.

V. CONCLUSION

A method has been described for estimating the available slope for a given aperture size, sidelobe level, and ripple factor. The results are helpful in the specification of sharp cutoff antenna patterns to reduce performance degradations caused by ground reflections and to evaluate trade-offs of the design parameters. The trade-off of high slope, high sidelobes, and high ripple versus low slope, low sidelobes, and low ripple is a key consideration in the design of some air traffic control systems. In some cases it has been determined that the benefit derived from a high slope factor near the horizon far outweighs some degradation caused by high ripple and sidelobes at large positive and negative elevation angles. This degradation at large elevation angles is generally not operationally significant, while performance near the horizon is usually critical. In other cases, such as MLS, low sidelobes at negative elevation angles are important.

VI. ACKNOWLEDGMENT

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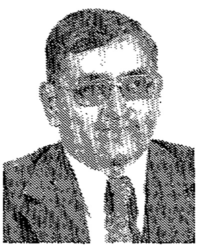

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