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1985 MAY 01
HAW/icf

Dr. Peder M. Hansen
Naval Ocean Systems Center
San Diego CA 92126

Subject: VLF Reports 1956-75

Dear Dr. Hansen:

In accordance with our previous conversations, I am pleased to send herewith a collection of VLF material which I prepared in 1956-75. This was all that was conveniently available.

If you are interested in anything particular, I should welcome an inquiry and I might be able to help you.

Yours sincerely,

H A Wheeler

H. A. Wheeler,
Chief Scientist

Hazeltine

Hazeltine and the Pursuit of Excellence

VLF Reports by H. A. Wheeler

1956-76

Collection for NOSC 850501

To Developmental Engineering Corp.

VLF ANTENNA NOTEBOOK - GENERAL

This is the first of a series of reports for collecting information relating to the design of VLF antennas, especially a high-power antenna for 15 Kc. Each report will be cumulative and will be assigned a block of numbers so that any page in this series will be identified by its page number. The report numbers are being assigned from the block of 301 to 309. The last digit of the report number will be the first digit of the 3-digit page number.

Contents | Page

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Index to This Series.

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Symbols and Units.

MKS rationalized units.

*Note: These quantities are RMS values unless otherwise noted.

f = frequency (cycles/second)

ω = $2\pi f$ = radianfrequency (radians/second)

λ = wavelength (meters)

$\lambda/2\pi$ = radianlength (meters)

μ_o = magnetivity (permeability) of free space (1.257×10^{-6} henries/meter)

ϵ_o = electrivity (permittivity) of free space (8.85×10^{-12} farads/meter)

k = ϵ/ϵ_o = electric ratio (dielectric constant) of medium (ground)

R_c = $\sqrt{\mu_o/\epsilon_o}$ = wave resistance of square area in free space (377 ohms)

σ = conductivity (mhos/meter)

δ = $1/\sqrt{\pi f \mu_o \sigma}$ = skin depth in conductor (meters)

R_s = $\sqrt{\pi f \mu_o/\sigma}$ = skin resistance of square area on surface of conductor (ohms)

H = magnetic intensity or current density (amperes/meter)*

E = electric intensity or potential gradient (volts/meter)*

I = current (amperes)*

V = potential (volts)*

P = power (watts)

C = capacitance (farads)

L = inductance (henries)

R = resistance (ohms)

X = reactance (ohms)

p = R/X = power factor ($\ll 1$)

a,b,c = dimensions (meters)

r = radial distance from center of vertical radiator (meters)

h = height (meters)

l = length (meters)

A = area (meters^2)

n = number

NB 77, p. 105-6.

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Information for 15 Kc.

$f = 15$ Kc

$\omega = 94.3$ K

$\lambda = 20$ Km $= 12.4$ mile

$\lambda/2\pi = 3.18$ Km $= 1.98$ mile

$\omega\mu_o = R_c(2\pi/\lambda) = 0.118$ ohm/m

$\omega\epsilon_o = (1/R_c)(2\pi/\lambda) = 0.833$ μ.mho/m or m.mho/Km

1 mi = 1.61 Km

1 ft = 0.305 m

Skin effect:

Copper	$\delta = 0.54$ mm	= .021 in,	R_s = .0320 milohm
Aluminum	0.70 mm	.027 in,	.041 milohm
Sea water ($\sigma = 4$)	2.0 m	6.7 ft,	0.122 ohm
Average ground ($\sigma = .002$)	92 m	300 ft,	5.5 ohms

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NB 77, p. 38, 80.

Example of VLF antenna properties:

Frequency	$f = 15$ Kc
Wavelength	$\lambda = 20$ Km
Radiated power	$P = 1$ Mw
Radiation resistance	$R = 0.1$ ohm
Radiation power factor	$p = .001 = 1$ mil
Reactance	$X = 100$ ohms
Current	$I = 3.16$ Ka
Voltage	$V = 316$ Kv
Capacitance	$C = 0.106$ μf
Effective height	$h = 159$ m $= 520$ ft
Effective area	$A = 1.90$ Km^2 $= 0.73$ mi^2
Inductance (to tune)	$L = 1.06$ mh

NB 80, p. 93

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(22) IRE, "Standards on Radio Wave Propagation, Definitions of Terms", 1942.

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kgs

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kgs

Definition and Test of Radiated Power.

One performance rating of the system is the amount of power radiated, as distinguished from dissipation in the antenna. There is no exact definition of this quantity, since there is no definite boundary between the antenna system and the surrounding space in which the radiated wave is propagated. For purposes of rating, we may define a boundary on some reasonable basis, and then charge to the antenna all losses within this boundary.

In the case of a very small dipole antenna (much smaller than the radiansphere) there is a definite field pattern at any distance beyond one radianlength. This pattern includes several terms, of which one is designated the radiation field and the others are designated the near field. We are interested in an antenna which is "small" (within half-radian hemisphere) at the lowest frequency of operation, and marginal in size at the highest frequency. The objectives are most difficult at the lowest frequency, so the radiation criterion becomes most important. Therefore the small-antenna principles will be applied.

We have a vertical electric dipole over a conductive ground plane. Its magnetic field has two terms in time quadrature, which are equal at all points on the radian hemisphere. Therefore this boundary divides the regions in which the near field and the far field respectively predominate. The near field represents the stored energy of the antenna condenser, so it is naturally associated with the antenna properties that affect the terminal impedance.

Substantially all of the losses that are reflected as resistance in the antenna impedance, occur within the radian hemisphere, mainly on the ground. Losses outside this hemisphere, since they are nearly imperceptible at the antenna, are naturally charged to the wave propagation that predominates in the outer region.

Therefore it is reasonable to define the radiated power as that which passes out of the radian hemisphere. This power can be tested in any region where the ground conditions approximate the ideal closely enough to assure the normal field pattern of a spherical wave.

The sea water is so good a conductor that its disturbance of the wave pattern is negligible over distances of several radianlengths. Therefore a valid test can be made at such distances in directions far removed from land areas. The Maine location is on a peninsula surrounded by water on three of its four sides, the land being within the radiancircle on the three sides.

It is proposed to measure the magnetic field along a radial in the southeast direction at 45° from the line of centers of the two halves of the antenna. This angle is chosen to minimize the very slight directive effect of the two-element array. Tests should be made from 1 to 3 radianlengths over the sea. There is a very small island between 1 and 2 radians, which might provide a location suitable for a monitor station.

The magnetic field along such a radial is easily analyzed for self-consistency and from its magnitude the radiated power is easily computed. It is subject to the least error caused by wave attenuation outside the radian hemisphere.

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VLF ANTENNA NOTEBOOK - PRINCIPLES

This is the second of a series of reports for collecting information relating to the design of VLF antennas, especially a high-power antenna for 15 Kc. Each report will be cumulative and will be assigned a block of numbers so that any page in this series will be identified by its page number. The report numbers are being assigned from the block of 301 to 309. The last digit of the report number will be the first digit of the 3-digit number.

Contents. Page

Requirements and Limitations of a Transmitter.

Radiated power is the end product of a transmitting antenna. It may be rated as the amount of power radiated beyond the radiansphere, in which case it is discounted by the principal power losses in the near field.

Modulation capability is an essential attribute of the radiated power. It involves many alternatives that are difficult to compare in utility.

Frequency bandwidth is one measure of the modulation capability. It permits proportionate speed of transmission. Alternatively, for the same speed of transmission, it provides excess bandwidth for more sophisticated types of modulation that increase the reliability or quality of reception.

Power limitation imposed by the antenna. The power that can be radiated from a VLF antenna is usually limited by the voltage that can be applied to it without excessive corona loss or actual arcing. The safe power rating should be made under the condition of rainfall, since such a condition must be tolerated frequently and it reduces the power capability by a factor of about 1/4. Infrequent extreme conditions, such as heavy icing or high winds, may be permitted to reduce the power capability below the rating.

Radiation power factor is the measure of the bandwidth that can be radiated efficiently by an antenna. Uniform response over a wider band can be obtained by increasing the resistance (heat) losses and providing proportionately greater power from the transmitter circuit. Or sideband pre-emphasis may be used ahead of the final power amplifier, in which case greater reactive power (volt-amperes) must be provided to drive the antenna over a wider band. In either case, the bandwidth can be increased indefinitely at the cost of increasing the available power.

Dissipation power factor is the measure of the heat losses in an antenna, its environment and associated circuits. The principal causes of such losses are (approximately in order of decreasing amount in practice):

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(1) ground resistance
(2) tuning inductors (and capacitors if used)
(3) dielectric losses (insulators, corona)
(4) wire resistance

In its effect on the antenna properties, each of these may be rated in terms of the power factor it contributes to the reactance of the antenna capacitance. At the cost of their power loss, such losses increase the useful bandwidth; they can be overcome by increasing the available power.

Efficiency of the entire antenna circuit is a measure of its ability to radiate power with less demand on the power amplifier; efficiency is not, in itself, an objective. Efficient radiation over a wider band requires increasing the radiation power factor by increasing the size of the antenna. This is a major consideration if the available power is only slightly greater than the required radiation.

Size of the antenna is the principal factor determining the radiation power factor. It is measured by the volume of the effective height times the effective area. The radiation power factor is proportional to this volume.

(29) HAW, "Fundamental limitations of a small antenna", Proc. IRE, vol. 35, p. 1479-84, Dec. 1947.

Effective height is the principal factor determining the radiation resistance. For a specified radiated power, the current is inversely proportional to the effective height.

Effective area is the theoretical area of a pair of condenser plates, separated by the effective height, that would provide the actual capacitance, assuming only a uniform field between the plates. For a specified radiated power, the voltage is inversely proportional to the effective area.

Conductor area is the principal factor determining the current that can be handled without corona. It is the total area of conductor in the aerial structure. It is best utilized by designing for uniform distribution of the charge. The permissible current is directly proportional to the conductor area.

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To Developmental Engineering Corp.

VLF ANTENNA NOTEBOOK - AERIAL SYSTEM

This is the third of a series of reports for collecting information relating to the design of VLF antennas, especially a high-power antenna for 15 Kc. Each report will be cumulative and will be assigned a block of numbers so that any page in this series will be identified by its page number. The report numbers are being assigned from the block of 301 to 309. The last digit of the report number will be the first digit of the 3-digit page number.

Contents | Page

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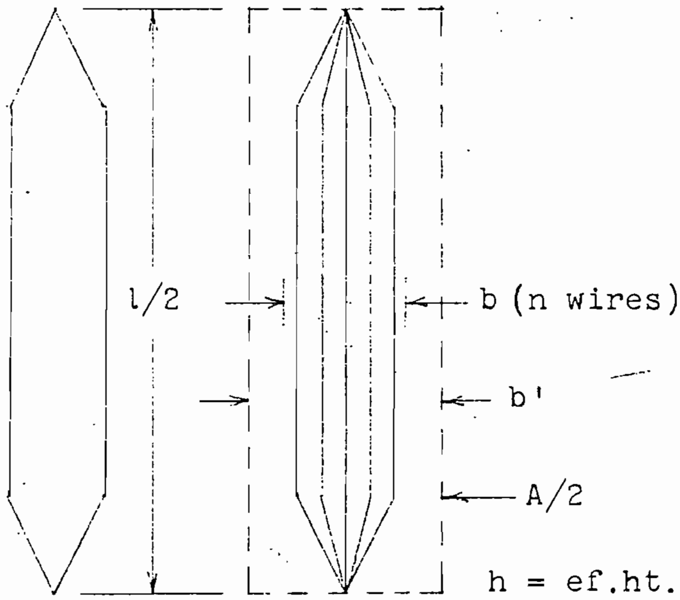
Computed Designs for Certain Requirements.



There have been derived a set of formulas which uniquely determine the principal dimensions of an antenna to meet certain requirements. Several cases have been computed and are reported here. (The formulas will be presented subsequently.) The common requirements are given first, leaving the height as the one independent variable in this series of examples.

Common requirements:

Frequency	f	=	15 Kc
Wavelength	λ	=	20 Km
Radiated power	P	=	1 Mw
Radiation power factor	p	=	.002 = 2 mils
Gradient on wires	E_a	=	0.67 Kv/mm (RMS)
Radius of wires	a	=	12.7 mm (dia = 1 in)
Total length of 2 sections	l	=	7.5 Km = 2 x 2.3 mi

Note: E_a is 1/4 the gradient for dry corona or 1/2 the gradient for wet corona; it is adjusted for the wire surface curvature, to 1.3 x 2.9 Kv/mm (crest).

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Note: l is chosen so $l/4$ (1/2 length of 1 section) is 1/8 wavelength at 20 Kc (enabling each section to be tuned up to this frequency by a central downlead).

Note: The form of the antenna is two long sections, each made of several parallel aerial wires and one or more downleads. The sections are parallel and are spaced several times their height; their interaction is ignored in these computations. Each section is tapered at the ends to maintain uniform gradient on the wires; this inherently removes the "end correction".

Effective height	-	600	750	900	ft
" "	h	183	230	275	m
Wire area	A_a	4900	4000	3300	m^2
Wire length	l_a	62	53	41	Km
No. of wires in grid	n	8.2	6.8	5.5	
Ef. area of grid	A	3.3	2.6	2.2	Km^2
Ef. width of grid	b'	440	350	290	m
Width of grid	b	260	67	11.5	m
Capacitance	C	.160	.102	.071	µf
Voltage	V	182	230	270	Kv

Note: The formula for the width b is based on uniform pitch and uniform charge, which is not realized. The pitch is graduated to obtain uniform charge and gradient, which decreases the required width of the grid. The average pitch is approximately b/n .

Note: The minimum requirements are exceeded and the performance improved if any of the computed dimensions are increased.

NB 80, p. 90-92.

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Effective Height and Antenna Current.

Assuming that the aerial wires over the ground act like a condenser free of inductance, the effective height is determined by the electric flux pattern between the aerial conductors and the ground conductors. Taking the terminals of electric flux lines, and noting the difference of height at the ends of these lines, their average difference of height is the effective height of the charge on the antenna. It is the principal factor determining the radiation resistance which relates the radiated power with the antenna current. It is assumed that the aerial system occupies a space much smaller than the radian hemisphere.

1580

$$R = \frac{1}{3\pi} R_c \left(\frac{2\pi h}{\lambda}\right)^2 = 40 \left(\frac{2\pi h}{\lambda}\right)^2 = 160\pi^2 \left(\frac{h}{\lambda}\right)^2 \qquad (1)$$

in which

R = radiation resistance (ohms)

h = effective height (meters)

(See p. 103 for standard symbols.)

$$P = RI^2 \; ; \quad I = \sqrt{P/R} \qquad (2)$$

in which

P = radiated power (watts)

I = antenna current (RMS amperes)

Expressing the current in terms of power and size,

$$I = \frac{\lambda}{2\pi h} \sqrt{3\pi P/R_c} = \frac{\lambda}{4\pi h} \sqrt{P/10} \qquad (3)$$

$$P = \frac{1}{3\pi} R_c \left(\frac{2\pi h I}{\lambda}\right)^2 = 160\pi^2 \left(\frac{hI}{\lambda}\right)^2 \qquad (4)$$

$$hI = \frac{\lambda}{2\pi} \sqrt{3\pi P/R_c} = \frac{\lambda}{4\pi} \sqrt{P/10} \quad \text{meter-amperes} \qquad (5)$$

This last product is a familiar rating of the power radiating ability of a transmitter.

For 1 Mw at 15 Kc: hI = 503,000 meter-amperes.

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Effective Area and Antenna Voltage.

In terms of the effective height and the capacitance of an antenna, the effective area of the aerial is the theoretical area of a pair of parallel plates separated by the effective height, that would have the same capacitance if the field were uniform and free from edge effects.

$$C = \epsilon_o A/h \tag{1}$$

$$A = hC/\epsilon_o \tag{2}$$

in which

C = antenna capacitance (farads)

A = effective area (meter^2)

h = effective height (meters)

The effective area is increased by edge effects but is decreased by the spacing between wires of an aerial wire grid.

The radiated power requires a voltage on the antenna capacitance, which is found to be inversely proportional to the effective area and independent of the height. The relation between current and voltage is:

$$I = VC\omega = \frac{VA\epsilon_o\omega}{h} = \frac{V}{R_c}\frac{2\pi A}{\lambda h} \tag{3}$$

in which

V = antenna voltage (RMS volts)

Equating this to formula (3) on p. 304,

$$V = \frac{\lambda^2}{4\pi^2 A}\sqrt{3\pi P R_c} = \frac{3\lambda^2}{2\pi A}\sqrt{\frac{PR_c}{12\pi}} = \frac{3\lambda^2}{2\pi A}\sqrt{10\ P} \tag{4}$$

$$P = \frac{V^2}{3\pi R_c}\left(\frac{4\pi^2 A}{\lambda^2}\right)^2 = \frac{1}{90}\left(\frac{2\pi A V}{\lambda^2}\right)^2 \tag{5}$$

$$AV = \left(\frac{\lambda}{2\pi}\right)^2\sqrt{3\pi P R_c} = \frac{3\lambda^2}{2\pi}\sqrt{10\ P} \quad \text{meter}^2\text{-volts} \tag{6}$$

This last product is a new rating of the power radiating capacity of an antenna subject to voltage limitations.

For 1 Mw at 15 Kc: $AV = 604\ \text{Km}^2\text{Kv}$

NB 81, p. 23-4.

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Effective Area and Antenna Voltage.

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For 1 Mw at 15 Kc: $AV = 604\ \text{Km}^2\text{Kv}$

NB 81, p. 23-4.

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Effective Volume and Radiation Power Factor.

The product of effective height times the effective area is the effective volume, which determines the radiation power factor.

$$p = RC\omega = \frac{1}{3\pi}\frac{Ah}{(\lambda/2\pi)^3} = \frac{8\pi^2 Ah}{3\lambda^3} \qquad (1)$$

in which

p = radiation power factor

R = radiation resistance (ohms)

A = effective area (meters^2)

h = effective height (meters)

Note: Compared with reference 1 below, this formula gives a value twice as great, because of the location adjacent to the ground plane.

$$Ah = \frac{3p\lambda^3}{8\pi^2} \quad \text{meter}^3 \qquad (2)$$

$$A = \frac{3p\lambda^3}{8\pi^2 h} \quad \text{meter}^2 \qquad (3)$$

For 15 Kc: Ah = 304 p Km^3

If either A or h is given, the other is determined by the performance specifications.

(1) HAW, "Fundamental limitations of a small antenna", Proc. IRE, vol. 35, p. 1479-84, Dec. 1947.

NB 80, p. 84.

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Corona Gradient on Round Wire.

The length and size of wire in the top of the antenna must be sufficient to handle the amount of charge associated with the capacitive current between the wire and ground. The charge density causes a proportionate electric gradient which must be held within the limit at which a corona discharge would occur and would cause excessive waste of power.

To make best use of the conductor area, the wire is disposed in a configuration that distributes the charge uniformly and makes the gradient nearly constant. There may be minor areas of low gradient but there must not be any areas of excessive gradient.

In a uniform field in air at a pressure of one atmosphere, a spark starts at a gradient of 2.9 Kv/mm or greater, depending on the frequency and the distance between electrodes (ref. 1). It is therefore safe to assume a threshold crest value of this amount or an RMS value of 2.05 Kv/mm.

On a round wire, a higher gradient on the surface is permitted before corona, because the gradient decreases with increasing radial distance (ref. 2). An empirical formula gives the experimental value of this effect at 60 cycles; it is believed that the same regime prevails at 15 Kc (ref. 1).

$$E_a = E_b \left(1 + \sqrt{a_1/a}\right) \quad (1)$$

in which

E_a = corona gradient on round wire (RMS Kv/mm)
E_b = 2.05 Kv/mm = breakdown in uniform field (RMS Kv/mm)
a_1 = 0.90 mm = wire radius (mm) on which $E_a = 2\ E_b$
a = wire radius (mm)

This effect obscures the increase of gradient on the fluted surface of stranded cable, if the outer strands have a radius less than a_1.

For comparison of wires differing in radius, it is convenient to formulate the effective radius and effective area of conductor that would be subject to corona starting at the standard gradient E_b.

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$$a' = a\,(1 + \sqrt{a_1/a}) = a + \sqrt{aa_1} \qquad (2)$$

in which

a' = effective radius of wire

The actual and effective conductor areas are:

$$A_a = 2\pi a\, l_a \;;\quad A_a' = 2\pi a' l_a \qquad (3)$$

in which

A_a = conductor area (for corona at E_a)

A_a' = effective conductor area (for corona at E_b)

l_a = length of wire

Example: a = 12.7 mm (dia = 1 in)

$a'/a = A_a'/A_a = E_a/E_b = 1.27$

For corona: $E_a = 1.27\, E_b = 2.60$ Kv/mm

The most severe condition that is common is wet weather. Water drops from the wire distort the surface and greatly facilitate corona. Some experience indicates that about 1/2 the gradient will cause corona. (Annapolis VLF antenna at 18 Kc, on wires of diameter 1 inch about 500 feet above ground, reported by W. W. Brown.)

In order to assure freedom from corona in a practical design, the following factors may be applied to the gradient:

1.27 for wires of dia 1 in

1/2 for water drops

1/2 for departure from constant gradient and any unknown factors such as surface contamination

$E_b = 0.51$ Kv/mm ; $E_a = 0.65$ Kv/mm

(1) HAW, "Nomogram for some limitations on high-frequency voltage breakdown in air", Wheeler Monographs No. 17, May 1953. (45)

(2) H. Pender, W. A. DelMar, "Electrical Engineers Handbook - Electric Power", McGraw-Hill, 4 ed., p. 14-31, 1949. (Corona gradient on round wire.) (32)

NB 80, p. 16, 86, 100.

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Conductor Area and Surface Gradient.

Assuming that the antenna charge is distributed uniformly over the aerial wires, the conductor area establishes a ratio between the capacitive current and the surface gradient. To establish this ratio, we conceive a theoretical parallel-plate condenser with constant gradient, having plates of area equal to the conductor area and of such separation that the capacitance is equal to that of the antenna.

$$C = \epsilon_o A_a / h_a \tag{1}$$

in which

C = antenna capacitance (farads)

A_a = conductor area of aerial wires (meters^2)

h_a = plate separation in condenser (meters)

The current in this capacitance is then

$$I = VC\omega = E_a h_a \frac{A_a \omega \epsilon_o}{h_a} = \frac{2\pi A_a E_a}{\lambda R_c} = \frac{E_a A_a}{60\lambda} \tag{2}$$

in which

I = antenna current (RMS amperes)

V = antenna voltage (RMS volts)

E_a = gradient on conductor surface (RMS volts/meter)

Equating this formula with (3) on p. 304,

$$E_a = \frac{\lambda^2}{4\pi^2 A_a h} \sqrt{3\pi P R_c} = \frac{3\lambda^2}{2\pi A_a h} \sqrt{10\ P} \tag{3}$$

$$A_a = \frac{3\lambda^2}{2\pi h E_a} \sqrt{10\ P} \tag{4}$$

Example: λ = 20 Km ; P = 1 Mw ; E_a = 0.65 Kv/mm (p. 308):

$A_a h$ = 930,000 m^3

This product determines the amount of wire required, for any height, to handle the power at the specified gradient. The gradient is set at a value estimated to be safely below the corona point.

NB 80, p. 11-12, 14, 46, 84, 99-100.

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Computation of Some Dimensions.

With the formulas given above, some of the dimensions of an aerial system can be computed very easily to meet certain requirements. It is necessary to specify the performance parameters and also some dimensions. The performance parameters are:

λ = wavelength
P = radiated power
p = radiation power factor
E_a = surface gradient on aerial wires

The dimensions to be specified are taken to be:

a = wire radius
h = effective height of aerial wires

Compute the following from the formulas noted:

p. 309 (4)	A_a	= conductor area
p. 308 (3)	l_a	= $A_a/2\pi a$ = length of wire
p. 306 (3)	A	= effective area
p. 305 (4)	V	= voltage
p. 304 (3)	I	= current
p. 304 (1)	R	= radiation resistance
p. 305 (1)	C	= capacitance
	X	= $1/C\omega$ = R/p = reactance

Some of these may be expressed more directly in terms of performance parameters:

$$V = \frac{2\pi h}{p\lambda}\sqrt{\frac{PR_c}{3\pi}} = \frac{4\pi h}{p\lambda}\sqrt{10\ P} \qquad (1)$$

$$C = \epsilon_o \frac{3p\lambda^3}{8\pi^2 h^2} \qquad (2)$$

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There remains the problem of distributing the length of wire in some pattern over an area large enough to give the specified effective area. Some formulas will be given further on, that are close enough and simple enough for estimating the size and supporting structures.

The voltage may be specified instead of the effective height. Unlike the primary performance parameters, the voltage would represent an upper limit. The corresponding upper limit on the height (if the other dimensions are held down to their requirements) is obtained from (1) above:

$$h = \frac{p\lambda V}{4\pi\sqrt{10\ P}} \qquad (3)$$

Example: λ = 20 Km ; P = 1 Mw ; p = .002 ; V = 250 Kv :

$$h = 252 \text{ m} = 825 \text{ ft}$$

This is within the range of examples on p. 303.

NB 80, p. 84-94.

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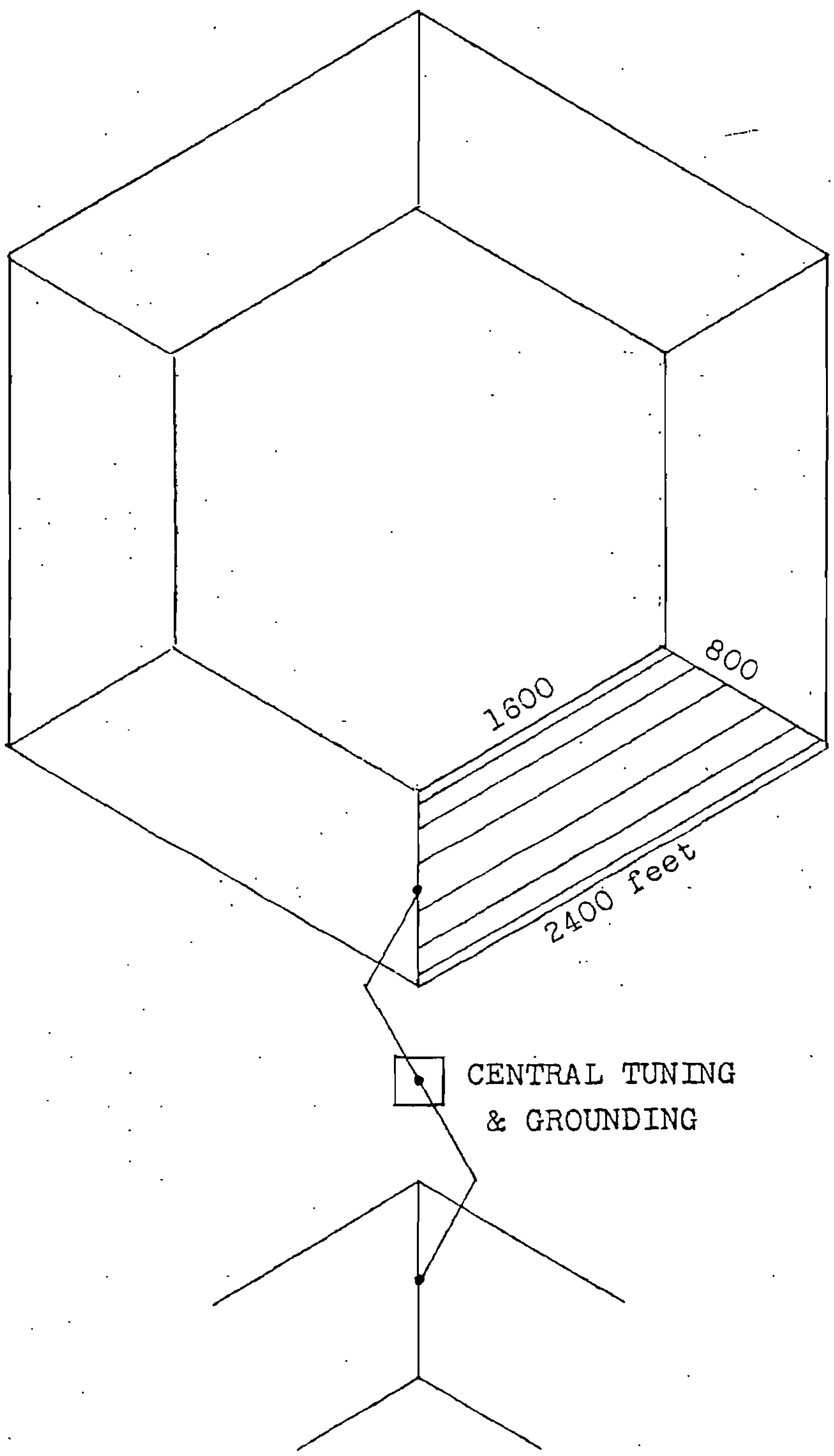
Triatic Ring

In this proposal, each independent half of the antenna is a hexagonal ring of triatic construction, supported by 6 outer towers and 6 inner towers. The example shown has the performance capabilities of one case described on p. 302-3, the case of 600 ft. effective height.



The triatic ring is a pattern of aerial wires which is made of a long triatic with its ends closed to form a polygon (hexagon). This arrangement is proposed to retain the principal benefits of the straight long triatic, while gaining some of the advantages of the six-pointed star and some advantages over either.

The example shown is intended to be comparable with two other plans under consideration. It is essentially similar to the pair of parallel triatics (p. 302-3). It has some other features of the pair of 6-pointed stars. Also it offers some advantages over either of these.

One objection to the straight triatic is the lateral wind pressure on the entire length of parallel wires at the same time. By closing the ends into a ring, the lateral wind pressure on these wires is reduced to 1/2 and becomes independent of the direction of the wind.

The outer towers carry much more than 1/2 the load. The triatic supporting cables at the outer towers must have counter-weighting or equivalent (such as a controlled winch). The supporting cables at the inner towers may not need such provisions if sufficient sag is provided.

Of all types considered, this one is easiest for electrical design. The environment of the parallel wires is nearly the same all around the ring. There are no "end effects" or bunching of wires to complicate the computations, which is most helpful in designing for uniform voltage gradient.

From one side of each ring, the total length of conduction path is the same as in the straight triatic, and only 1/2 as great as in the star. Like the straight triatic, the ring could be adapted for the central tuning and grounding of a pair of structures forming both halves of the complete antenna.

If the central tuning is used, the voltage is slightly greater on the far side of each ring. Uniform gradient on the parallel wires may be attained by decreasing the wire spacing on the far side and increasing it on the near side. The capability of the suspension insulators might be adapted to this condition by slightly changing the number of insulators in each string.

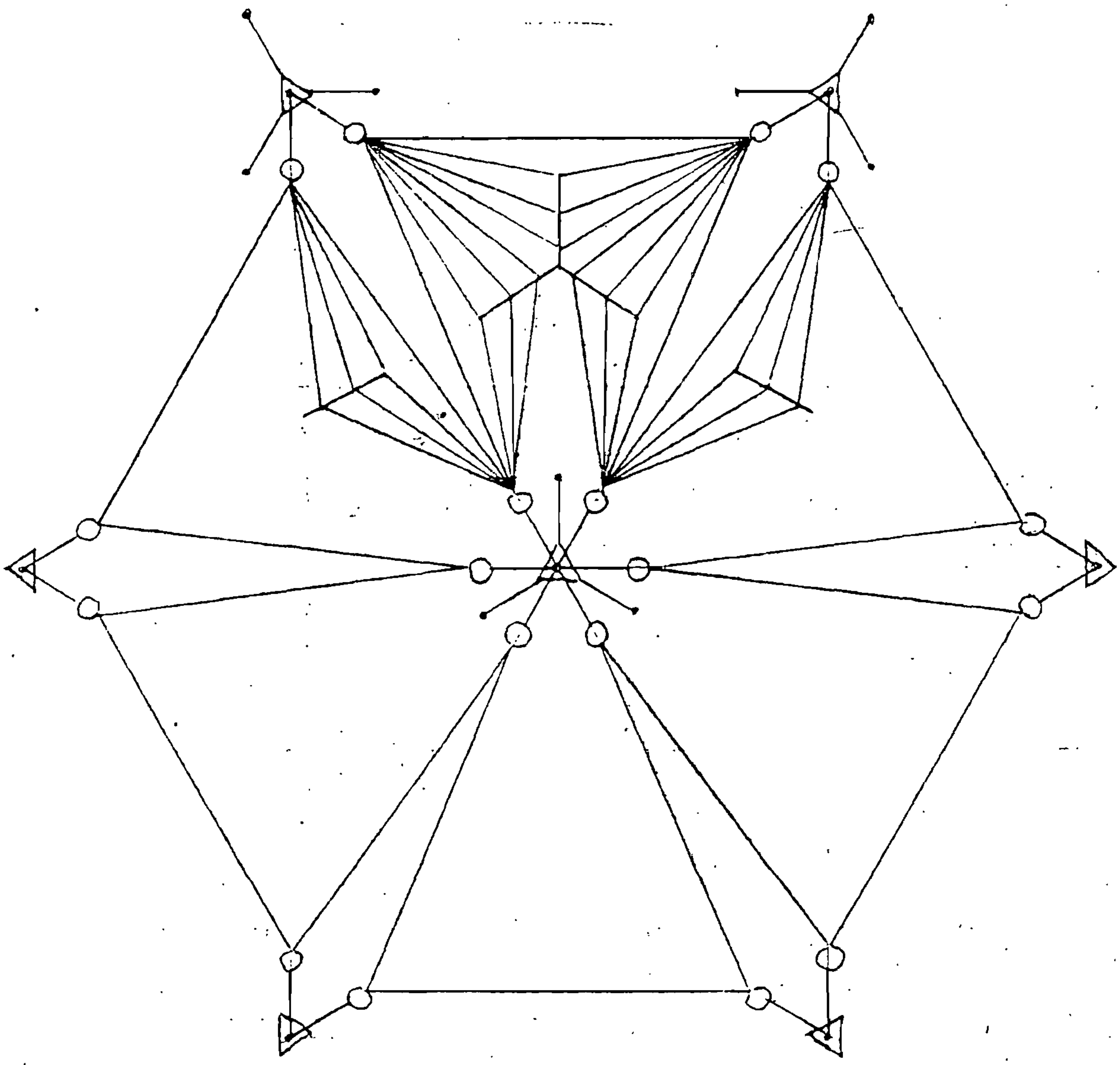
If multiple tuning is preferred, the most elementary form would require an extra downlead on the far side of each ring. This tuning may be designed to take about 1/2 the current at the lowest frequency (15 Kc) and need not be adjustable; also it might be so designed as to obviate the need for any de-icing circuits at its location. Like any multiple tuning, it would complicate the pattern of wires in the ground system.

To minimize the shadowing toward the nearby radar station, the towers should be so located that the radar never sees more than 2 towers in line (see Report 310). The ring arrangement reduces to 1/2 the total shadowing effect of the aerial wires, because their component of length that causes shadowing is only 1/2 the total length. (This is also true of the star.)

As compared with the straight triatic, the obvious disadvantage of the ring triatic is the difference between inner and outer dimensions of the ring, causing a proportionate difference of tower loading and wire spans. Since the total loading and the average span remain about the same, it is expected that the total cost of construction would also remain about the same. There would be a large saving if only the outer cables need counterweighting.

This design is attractive for several reasons and may offer the best combination of features as compared with the alternatives under consideration.

Hexagon (6-leaf cross).



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The 6-leaf cross is a form of hexagon pattern for distributing the aerial wires over the flat top. It is proposed to use two such structures to form the independent halves of the entire antenna. Each requires 7 towers, making a total of 14 towers.

The objective of any such structure is to spread the required length of wire over an area sufficient to give the required capacitance. The 6-leaf cross is intended to make the best use of a hexagonal area, covering all except the opening for the central tower.

This structure is intended to offer some constructional features with the intent of reducing the cost. The principal feature is the reliance on direct suspension of all spans rather than cross-catenary suspension. This increases the total length of the span but otherwise simplifies the problems of design. All spans are nearly equal.

A structural feature of the reentrant perimeter is a flexibility of the wire pattern which reduces the transfer of loading from any one span to other spans. This is intended to ease the design for wind loading. As in any pattern of 3 or more sectors of symmetry, the cross-wind loading is only half effective on the total length of wire.

The guy wires on the towers are easily arranged to leave clear space for lowering the flat top.

One objective is economy in the number of towers. The combination of 6 outer towers and one central tower in each of two structures requires a total of 14 towers. None of the towers has any extreme requirements.

The locations of the towers are favorable for electrical performance. The outer towers are further than usual from the active grid of wires. The inner tower is in a region of reduced electric field.

For most capacitance and power factor, and least voltage, the required length of wire should be spread over the most area. This objective has to be compromised in favor of economy, so it is pursued only so far as necessary to meet the needs for increasing the power factor and decreasing the voltage. Greater area is covered with fewer wires in parallel in each span.

This pattern was devised by the writer for an umbrella structure with only a central tower. The diagram would be modified to

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insulate the tower and to use it as the downlead. The plan is well suited for such a structure, which however seems to offer no net advantage over the flat top here proposed.

The pattern of wires in the flat top is basically a few wires in parallel around the perimeter of a polygon. The parallel plan is recognized as good for freedom of design in multiple tuning and in heating circuits. The perimeter is unfortunately long, about 2.4 times that of the hexagon, but this is competitive with some other schemes for 7 towers (such as the 6-pointed star) since it requires less area of land for the same perimeter.

The diagram shows roughly optimum proportions of dimensions. The supporting cables from the towers are about 1/4 the length of the active span. An example of actual dimensions will be given below.

Adjacent wires are nearly parallel over most of the active area of the grid. This enables rough computation of the charge distribution, with the objective of uniform gradient over the wires. This objective is served by parallel wires as contrasted to crossed wires. The bunching of the wires at the vertices holds down the gradient where it would tend to be greatest on a continuous sheet of conductor. The spacing should be nearly uniform except on the outer sides of the polygon, where the spacing should decrease toward the outer edge.

There are several attractive alternatives for downleads and tuning. The simplest is a pair of downleads from two inner vertices on opposite sides of the central tower, brought down to a single tuner; this is severely restricted in tuning range. Another plan is 2 or 3 downleads from outer vertices equally spaced around the perimeter.

This pattern requires a high-voltage insulator at every vertex, as shown in the diagram: 18 for one hexagon or 36 for the entire antenna. Additional low-voltage insulators, not shown, are required for the heating circuits.

Each high-voltage insulator is located near the vertex, because any appreciable length of supporting cable at high voltage would be subject to corona.

The following example is based on rough computations for radiation of 1 Mw at 15 Kc with 200 Kv and a power factor of .004.

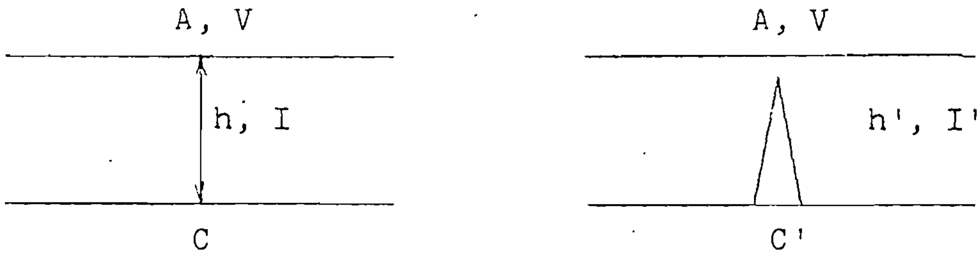
ew

Effective height:	200 m	=	660 ft.
Flat-top height:	250 m	=	820 ft.
Radial distance to outer towers:			2350 ft.
Length of active span:			1600 ft.
Length of supporting cable:			400 ft.
Wire diameter:			1 in.
Length of wire:	69 Km	=	230,000 ft.
Length of wire required if uniform gradient:	58 Km	=	190,000 ft.
Capacitance:	0.13 µf		

NB 81, p. 11, 63; NB 80, p. 104.

ew

Influence of towers on effective height.



(a)   (b)

It is well known that any supporting towers increase the capacitance and decrease the effective height (see definition p. 304). Two theorems have been proved by the writer for establishing some relations between these two effects.

The first theorem is an exact relation in an idealized case represented in the diagrams. The aerial structure is a condenser, between the ground and an elevated sheet conductor so extensive that some of the interior space is substantially shielded from the exterior. The top is shown as a plane conductor parallel to ground, which is the simplest case. The radiation from such a structure is associated with the external electric field, which is determined by the effective area (A, see definition p. 305) and the voltage (V).

Any object in the shielded space has no effect on the external field and the radiated power, but may affect the capacitance (C) and hence the current (I) associated with the voltage. Keeping the voltage the same, the current is proportional to the capacitance; keeping the power the same, the effective height is inversely proportional to the current and hence to the capacitance. These are expressed as follows, in terms of the formula for capacitance.

$$\epsilon_o A = hC = h'C' \quad (1)$$

$$h'/h = C/C' < 1 \quad (2)$$

$$\Delta h/h = -\Delta C/C \quad (3)$$

ew

in which

ϵ_o = electrivity (permittivity) of air

A = effective area

C = capacitance of sheet and ground

C' = capacitance increased by insertion of extra conductors in shielded space

h = effective height of sheet above ground

h' = effective height decreased by insertion of extra conductors in shielded space

Δ = small increment caused by insertion of extra conductors

Equation (2) states the common rule for this effect, so we now see the conditions under which this rule is valid. This rule states that the effective height changes in the inverse ratio of the capacitance. In one extreme, the rule applies to a lumped capacitor connected between the planes. While the effective area is unchanged, so the same voltage is still capable of radiating the same power (p. 305), it does this with greater current and lesser power factor.

In practice, we may consider the top sheet to be replaced by a wire grid occupying the same area, with the insertion of a grounded supporting tower under the center. Without the tower, the wire grid is taken to provide capacitance equal to a fraction (m) of that of a sheet on top. The tower causes the same reduction of effective height in both cases, but the change of capacitance of the wires is only a fraction (m^2) as great as the change for the sheet. For a small relative change of capacitance, we have the relation,

$$\frac{\Delta h'/h'}{\Delta h/h} = \frac{1}{m}\frac{\Delta C'/C'}{\Delta C/C} = m < 1 \quad ; \quad \Delta h'/h' = -\frac{1}{m}\Delta C'/C' \qquad (4)$$

in which

m = C'/C above ground = ratio of capacitance of wire grid over that of sheet

C' = capacitance of wire grid and ground

h' = effective height of wire grid

This means that the relative change of effective height is greater for a wire grid than it is for a sheet on top. This becomes a general rule, applicable to practical problems.

A second theorem gives a comparison of any two regions inside or outside the shielded space. It is strictly applicable to small extra conductors, but the concept is applicable to large conductors such as peripheral supporting towers that are naturally outside the shielded space. The electric field (E) outside is less than the field (E_o) inside the shielded space, so their ratio is less than unity ($E/E_o < 1$). The increase of capacitance caused by a small conductor is proportional to the square of the electric field at its location because it is caused by a two-way reaction. On the other hand, the decrease of effective height is caused by the electric dipole moment induced on the conductor, which is a one-way reaction and hence is directly proportional to the electric field. From these two laws and equation (3) we can write the resulting relation between relative changes of capacitance and effective height.

$$\frac{\Delta h/h}{\Delta C/C} = - \frac{E/E_o}{(E/E_o)^2} = - E_o/E \tag{5}$$

in which

E_o = electric field in shielded space
E = electric field at location of extra conductor

Therefore a peripheral supporting tower causes a relative change of effective height greater than the relative change of capacitance. A short distance outside of the area occupied by the wires, the electric field falls to about 1/2 its value in the center of the area, so the ratio given by equation (5) is of the order of two. (This has been observed recently in model tests on this project.)

The second theorem is restricted by the fact that only the vertical component of induced dipole moment decreases the effective height. The theorem could be stated more generally, but the simple equation (5) is valid for two cases of greatest interest. It is valid for conductors of any shape located near the ground where the field is vertical. Or it is valid in any location for vertical conductors of relatively small transverse dimensions, such as vertical towers. In these cases, the induced dipole moment is vertical.

Each of these two theorems gives a factor by which the relative change of effective height exceeds that of capacitance. Both are applicable to supporting structures from which the aerial wires are insulated.

Downleads are comparable in some respects. If located in the shielded space under a sheet conductor, they obey the same rules. Otherwise, it is found that the relative change of effective height is less than that of capacitance.

First, if the downlead is connected with a grid of wires, it contributes somewhat to the radiation for the same voltage, because it is partially coupled with the outer space through the wires. This increase of effective area is reflected in a lesser decrease of effective height.

Secondly, the downlead is usually located in a space where the electric field varies considerably from top to ground. In this case, the average value of the field is slightly less than the RMS value, so this relation would further cause the relative change of effective height to be slightly less than that of capacitance.

It is concluded that downleads are subject to rules that are generally opposite to those for supporting towers in the usual structures.

Supporting towers for a grid of wires reduce both effective height and effective area, which combine to make the effective volume. On the other hand, the downleads may cause opposite changes of effective height and effective area that would leave the effective volume about the same.

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Design for Specified Voltage and Gradient.

The two principal limitations on the radiated power are the voltage rating of the suspension insulators and the corona gradient on the aerial wires. It is conjectured that the cost is minimized if both of these limitations are effective at the same power level. To this end, some of the relations given above are expressed explicitly in terms of these quantities, and some further concepts are introduced.

If there is a value that must be met or exceeded by the radiation power factor (p), this places a proportional requirement on the effective height (h). (See p. 304, 306.)

$$h = \frac{p\lambda V}{2\pi}\sqrt{\frac{3\pi}{PR_c}} = \frac{p\lambda V}{4\pi\sqrt{10\ P}} \tag{1}$$

It may be that the cost is minimized at a greater height, which is permissible.

If there is no requirement on the power factor, its value or the height may be taken as an unrestricted variable for minimizing the cost while meeting all other requirements. The height is so regarded in the following discussion. Then we have the resulting value of power factor.

$$p = \frac{2\pi h}{\lambda V}\sqrt{\frac{PR_c}{3\pi}} = \frac{4\pi h}{\lambda V}\sqrt{10\ P} \tag{2}$$

$$h/p = \frac{\lambda V}{2\pi}\sqrt{\frac{3\pi}{PR_c}} = \frac{\lambda V}{4\pi\sqrt{10\ P}} \tag{3}$$

This quotient is constant for the present purpose.

The required effective area is determined by the specified upper limit of the voltage (p. 305).

$$A = \left(\frac{\lambda}{2\pi}\right)^2\sqrt{\frac{3\pi PR_c}{V^2}} = \frac{3\lambda^2}{2\pi V}\sqrt{10\ P} \tag{4}$$

The conductor area is determined by the height and the corona gradient (E_a). (See p. 309.)

$$A_a = \frac{\lambda^2}{4\pi^2 hE_a}\sqrt{3\pi PR_c} = \frac{3\lambda^2}{2\pi hE_a}\sqrt{10\ P} \tag{5}$$

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$$A_a h = \frac{\lambda^2}{4\pi^2 E_a}\sqrt{3\pi P R_c} = \frac{3\lambda^2}{2E_a}\sqrt{10\ P} \qquad (6)$$

This product is constant for the present purpose. The required length of wire is

$$l_a = A_a/2\pi a \qquad (7)$$

in which

l_a = length of wire

a = radius of wire

There is a simple relation that applies for the equal limitations of voltage and corona:

$$\frac{A_a}{A} = \frac{V}{hE_a} \ll 1 \qquad (8)$$

This is the "filling factor" of the aerial wires; here it is much less than unity.

An interesting shape factor is the following "spreading ratio":

$$\frac{A}{h^2} = \frac{Ah}{h^3} = \left(\frac{\lambda}{2\pi h}\right)^2\sqrt{\frac{3\pi P R_c}{V^2}} = \frac{3\lambda^2}{2\pi h^2 V}\sqrt{10\ P} \qquad (9)$$

This is the number of "height cubes" in the effective volume.

The capacitance (C) and its reactance (X) are as follows.

$$C = \epsilon_o A/h \qquad (10)$$

$$X = \frac{1}{\omega C} = R_c\frac{\lambda h}{2\pi A} = \frac{60\ \lambda h}{A} \qquad (11)$$

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The following is an example based on these formulas.

	λ	=	20 Km (15 Kc)
	P	=	1 Mw
	V	=	180 Kv (RMS)
	E_a	=	0.87 Kv/mm (RMS)
(4)	A	=	3.35 Km^2
(6)	$A_a h$	=	694,000 m^3
(3)	h/p	=	90.6 Km = 90.6 m/mil

From these constants, the independent variable (h) may be chosen for least cost. It is here given a value typical of recent studies.

	h	=	160 m = 525 ft
(2)	p	=	.00177 = 1.77 mils
(5)	A_a	=	4330 m^2
	a	=	12.7 mm (wire dia 1 in)
(7)	l_a	=	54.3 Km = 178 Kft = 33.6 mi
(8)	A_a/A	=	.00129
(9)	A/h^2	=	131 = $(11.4)^2$
(10)	C	=	0.185 µf
(11)	X	=	57.3 ohms
	R	=	0.101 ohm
	I	=	3.15 Ka

This example yields a set of values that are typical of a balanced design for the present purpose.

Returning to a question of cost, let us consider the probable result of a balanced design, that is, one operating simultaneously at the safe limits imposed by insulator voltage and corona gradient.

If the effective height is invariant, it is easy to arrive at some conclusions. The required conductor area is determined by the gradient and the required effective area is determined by the voltage.

Operating below the safe voltage requires that the conductor wire be spread over a greater area, which increases the cost of suspension.

Operating below the safe gradient requires that the conductor area be increased while retaining the same effective area; this means that the greater amount of wire may be spaced more closely to cover a smaller actual area. Here there are two opposing influences, so there is no definite rule whether the cost would be increased by "unbalancing" the design in this respect. It is conjectured that the cost of increasing the amount of wire (with proportionate wind and ice loading) exceeds the saving by closer spacing.

Variation of height would have some influence on both of these design limitations. However, if a balanced design is most economical at any particular height, its benefits are retained at whatever height may serve to minimize the cost. Here it is conjectured that the cost has a wide minimum with variation of height if designed for maximum safe values of voltage and gradient.

NB 81, p. 117-27.
Report 311.

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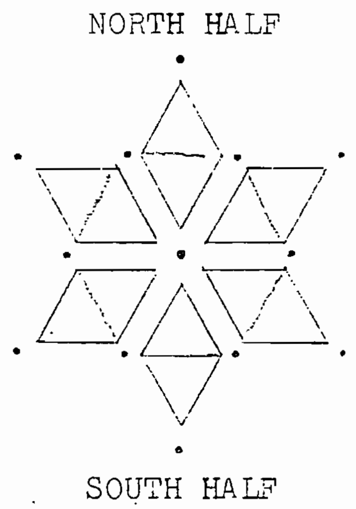
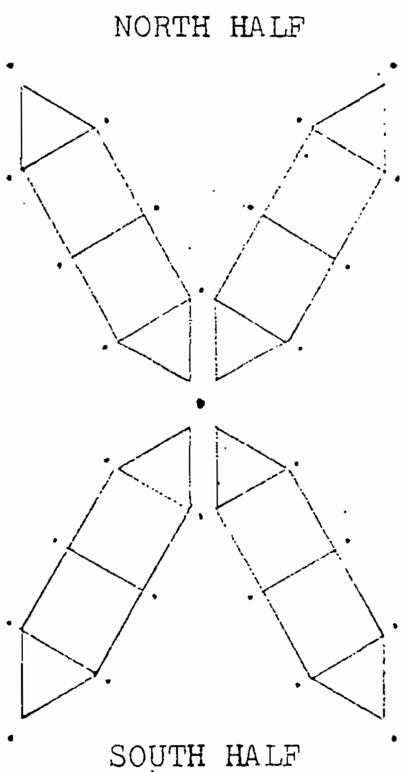
Radial Triatic Patterns.

(a)



6 arms	8 arms
12 sections (shown)	16 sections
12 + 1 towers	16 + 1 towers
24 insulators	32 insulators
12 + 6 sections	16 + 8 sections
24 + 1 towers	32 + 1 towers
36 insulators	48 insulators

(b)



4 arms (paired)

8 + 8 sections (shown)
26 + 1 towers
32 insulators

8 + 4 sections
18 + 1 towers
24 insulators

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The radial triatic pattern of aerial wires has been the subject of recent studies at DECO leading to more appreciation of its advantages over other forms. It has been intended to use two separate radial triatics as the independent halves of a single aerial system. It is here proposed to use a single triatic pattern about a single center, but divided into two independent halves. This would enable the use of a central ground, at least for the lower frequencies which impose the most severe requirements on the ground system. After a brief discussion of the "star" pattern now under study, the "X" pattern will be proposed to fit the oblong space available on the peninsula. It represents the least departure from the two parallel triatics (p. 302).

In general, the radial triatic retains most of the simplicity of the single triatic. Cross-catenary supports have mechanical simplicity and a background of experience in previous designs. Parallel wires are adapted for graded spacing and for heating circuits.

The mechanical design is eased by dividing the flat top into a greater number of shorter arms. The multiple duty of some of the towers reduces the required number of towers and balances the moments of some components of their loading.

The single-center radial triatic, with its simplicity of central tuning and grounding, offers the greatest efficiency for the lowest frequency where most difficult to accomplish. It localizes all feed points so no long feed lines are needed. It is even more attractive if the total cost and size have to be reduced.

The star form of radial triatic (a) has a number of like radial arms, each arm having the shape of a diamond with two triangular sections, or elongated by the insertion of one or more rectangular sections. Six arms is the least number that is attractive, because it is preferable to make each arm rather narrow so the wires are nearly parallel. Eight arms is a shape that may be better toward this objective, and perhaps the best compromise. Several alternative patterns are tabulated. If the loading of each insulator is to be held within a specified limit, a greater area can be covered by increasing the number of arms and the radial span while decreasing the number of wires in each arm. The single star of equal arms and angles is poorly adapted for utilizing the oblong space available.

ew

The "X" form of radial triatic (b) is a variation of the single-center star form, with the object of providing two independent halves to utilize an oblong area. This pattern is also a minor change from the two parallel straight triatics and retains most of their good features. Furthermore, the division of each long straight triatic into two parts greatly eases the mechanical design. The allotment of the upper and lower "V" patterns to the independent halves serves to reduce their intercoupling.

The single-center design offers these main features for electrical performance.

(1) A central feed location for the transmitter house, suited for keying reactors and all heating circuits, avoiding long feed lines.

(2) A central ground system of simple radial wires to be used exclusively for the lower frequencies, where most needed, and perhaps for all frequencies.

The simplest tuning is single tuning of each half to the central ground. This is feasible for the lower frequencies (say 14-20 Kc) by series inductors (including variable inductors and keying inductors). Unification of these inductors for each half offers least cost and least losses.

The higher frequencies (say 20-30 Kc) probably require a choice between series capacitors or multiple tuning. These alternatives will be discussed briefly.

Series capacitors are not in common use in antenna circuits such as these; in fact, the writer does not know of any instance in all the high-power VLF transmitters he has been studying. Therefore a new type of condenser assembly might have to be developed, but no inherent difficulties are perceived. It would be a massive component, perhaps comparable with the inductors in size. The series capacitor would present a problem of grounding the antenna for lightning, but a parallel choke coil could be designed for this purpose, considerably smaller than the tuning inductor. Probably the series capacitor would be connected between the downlead and the inductors, so the latter would be grounded. The capacitor might be switched in several sections to reduce the required range of the inductors. If there is sufficient time to develop the series capacitor, it is believed to be more promising than multiple tuning.

Multiple tuning of this X-form would be accomplished by an extra downlead at the outer end of each of the four radial arms. This point is chosen because it can be a junction of all wires that is neutral in the heating circuit, so each inductor is simply grounded. This expedient is available if the series capacitor should prove unattractive or unavailable within the limited time. Each point of multiple tuning would require a sub-center of grounding that could be much less elaborate than the main center, because its share is only 1/8 of the total current (compared with 4/8 or 1/2 in the main center). Furthermore, multiple tuning might be used to the full extent only at the higher frequencies, where the grounding requirements are less severe. In any case, the multiple-tuning inductors would require only a few fixed values, perhaps 3 or 4 values (of which one might be open-circuit).

If the aerial were to be greatly reduced in size, it might develop that neither series capacitors nor multiple tuning would be needed.

NB 81, p. 115-6, 118, 128.

ew

Shape of Aerial for Least Cost.

If the performance is specified, there is some design that gives this performance with least cost. In the antenna system, the principal variables are (1) the size and shape of the aerial structure and (2) the requirements imposed on the accessories, including the ground system and tuning inductors. We wish to outline the factors that vary with shape and lead to a design for least cost. As an example of these factors, an aerial system is assumed to have the most economical one of the usual patterns; then the effective height (h) is taken as the independent variable and other structural dimensions are allowed to vary in such a manner as to retain the same performance.

The performance is specified in terms of these quantities:

λ = wavelength
P = radiated power
V = antenna voltage (limited by insulators)
E_a = wire gradient (limited by corona)
Radiation efficiency (say 0.50)

While not strictly a performance parameter, the wire size is held constant; this determines its outer surface but its inner density is variable with the tensile strength required. The parameters are to vary in such a relation that the specified voltage and gradient are both realized (the principle of the balanced design, p. 323-6).

After a brief statement of the principal theoretical relations that form the basis for this study, there will be an attempt to establish the laws that determine the shape for least cost. About half of the proportionalities have an exact basis and the rest are approximations that are believed to indicate the trend correctly.

The basic rules are the constants determined independently by V and E_a:

Effective area: $A = cst$ (1)
Conductor area x height: $A_a h = cst$ (2)

kgs

The actual area covered by the flat top varies with shape because of the fringing field and the wire spacing. The two areas are related as follows:

$$\frac{\text{Covered area}}{\text{Ef. area }(A)} \doteq 1 \quad \text{for } h \text{ very small} \tag{3}$$

$$< 1 \quad \text{for } h \text{ moderately small}$$

$$> 1 \quad \text{for } h \text{ large}$$

This progression reflects first the increasing fringing field and then the increasing separation of the wires. The concept of covered area is significant for many or several parallel wires, and fails for a single long wire.

Since the wire radius (outside) is constant:

$$\text{Length of wire:} \quad l_a \propto 1/h \tag{4}$$

Since the covered area varies slowly with shape, we have roughly:

$$\text{Wire separation} \propto h \tag{5}$$

The efficiency is a relation between the aerial on one hand and all accessories on the other hand. The latter include the heating circuit (for preventing or melting ice on the wires), the ground conductor system, the tuning and keying inductors, any series capacitors and any feed lines. With the exception of the heating circuit, all of these become cheaper if h is greater, in a manner that will be mentioned briefly.

The radiation power factor is proportional to the effective volume and hence the effective height:

$$\text{Radiation power factor:} \quad p = 1/Q \propto Ah \propto h \tag{6}$$

$$Q \propto 1/h \tag{7}$$

The power factor of inductors and capacitors must be held much less than that of radiation, so these components are cheaper if h is greater. In the keying inductor, the stored energy is proportional to Q and hence requires a proportional amount of iron in the core.

kgs

For efficiency, the ground system requires such a concentration of conductors that its resistance is much smaller than the radiation resistance:

Radiation resistance: $R \propto h^2$ (8)

Other relations of incidental interest are the following:

Antenna current: $I \propto 1/h$ (9)
Capacitance: $C \propto 1/h$ (10)
Reactance: $X \propto h$ (11)
Frequency bandwidth: $pf \propto h$ (12)

This last quantity is a measure of on-off keying speed and also of tolerance of detuning of the antenna circuit (especially during frequency-shift keying, FSK).

To enable some simple conclusions, the change of shape is quantized in terms of similar cells forming a basic element of the aerial system. It is assumed that there is a shape of cell that is least costly for obtaining its performance. The shape is described by the pattern of towers and wires, and the ratios of all length dimensions over the effective height. These ratios include the tower heights and distances, the lengths of spans of the wires, and the effective height of the flat top of wires. The number of parallel wires in the top is a separate variable not embraced in the shape; it will give a required freedom of design in relations that are valid for several or many wires, but not for too few wires (one or two). Except for the wires (constant radius and variable number), a cell retains the same shape when combined in various sizes and numbers.

A <u>single cell</u> has these relations:

Effective height	$= h$	(13)
Height of towers	$\propto h$	(14)
Distances between towers	$\propto h$	(15)
Lengths of wire spans	$\propto h$	(16)
Covered area	$\propto h^2$	(17)
Effective area	$\propto h^2$ (approx.)	(18)
Effective volume	$\propto h^3$ (approx.)	(19)

The last two relations are strictly valid only if all the cells are separated sufficiently to be substantially uncoupled, or are assembled in groups (super-cells) complying with this rule. In practice, it is more likely that several cells will be located close enough to share some of the towers, in which case the last two relations may fail by a substantial margin.

A typical single cell has horizontal length and width several times the effective height, so its covered area is perhaps 4 to 16 times the height-square (h^2).

The <u>number of cells</u> varies with the height and resulting shape of the group of cells forming the aerial system. Since the effective area of the system is to be held constant (1):

$$\text{Number of cells:} \quad n \propto A/h^2 \propto 1/h^2 \text{ (approx.)} \tag{20}$$

This is subject to the same uncertainity as (18) and (19). The strict rule is, that the number of cells is the number required to keep the effective area (A) constant for various heights in any particular arrangement of cells. Proximity of cells causes a greater variation of number with height, so the negative exponent of h is greater than 2.

The electrical properties of the <u>wire</u> are governed by (2), (4), (5) above. The <u>mechanical load and strength</u> are also proportional to the <u>density</u> inside the wire, whether hollow or filled with steel cable. By the rules of the single cell, the length and sag of each span of wire are proportional to the height. The material of the wire must be strong enough to carry several times

kgs

its own weight in the longest span, regardless of the density of the wire. However, the wind and ice loading are proportional to the length, so the strength and hence the density should be proportional to height. (There is an upper limit as the wire becomes filled with steel cable.)

Density of wire	$\propto$ h (approx.)	(21)
Total weight of wire	$\propto$ cst (approx.)	(22)
Weight of wire per cell	$\propto h^2$ (approx.)	(23)

The various forces on the wire and the heating power are proportional to the area, or here to the length exposed:

Total wind force on wire	$\propto 1/h$	(24)
Total ice weight on wire	$\propto 1/h$	(25)
Total heating power	$\propto 1/h$	(26)

Wind force per cell	$\propto$ h	(27)
Ice weight per cell	$\propto$ h	(28)
Heating power per cell	$\propto$ h	(29)

All of these relations are simplified approximations.

If the area is covered by wires that are (at least roughly) parallel, the wire separation is proportional to covered area over length of wire:

Wire separation $\propto$ h (approx.) (30)

The wire loading on the towers is related to the moments of the various forces times the height. These moments have the following total values for the aerial system:

Total moment of wire weight	$\propto$ h	(31)
" " " wind on wire	$\propto$ cst	(32)
" " " ice weight	$\propto$ cst	(33)

The corresponding moment per cell is:

kgs

Cell moment of wire weight $\propto h^3$ (34)

" " " wind on wire $\propto h^2$ (35)

" " " ice weight $\propto h^2$ (36)

<u>Each tower</u> is first designed to support its wire load, and is then strengthened to carry the load of its own structure (self-load). The rules to be given apply to either self-supporting rigid towers or guyed thin towers.

For each tower, the cross-sectional area of the legs and guy wires is proportional to the horizontal forces of the wire loads of one cell. The corresponding volume and weight of these structural members is also proportional to their length or the height:

Tower cross-sectional area $\propto h$ to h^2 (37)

Tower volume $\propto h^2$ to h^3 (38)

The self-load per tower is caused by its weight and its exposed surface. The latter determines the wind force and the ice weight. The amount of strengthening of the tower is related to the strength required to carry the wire loads. The weight load of the volume (38) requires an increment of cross-sectional area and volume:

Δ tower cross-sectional area $\propto h^2$ to h^3 (39)

Δ tower volume $\propto h^3$ to h^4 (40)

For the other loads it is noted how the exposed surface of the towers varies with wire loads:

Tower cross-sectional width $\propto$ area$^{1/2}$ $\propto h^{1/2}$ to h (41)

Tower surface area $\propto h^{3/2}$ to h^2 (42)

The weight load of ice on the tower, which is proportional to the surface area, requires further increments:

Δ tower cross-sectional area $\propto h^{3/2}$ to h^2 (43)

Δ tower volume $\propto h^{5/2}$ to h^3 (44)

kgs

The wind force on the tower, also proportional to the surface area, causes a moment which requires further increments following the same laws of variation (43), (44). It is noted that the increments required by self-load vary with height somewhat more rapidly than the initial values required by wire load. All of these loads have variations within the ranges:

Tower cross-sectional area $\propto h$ to $h^3 \propto h^2$ (say) (45)
Tower volume $\propto h^2$ to $h^4 \propto h^3$ (say) (46)

The <u>number of towers</u> for various heights and cell sizes is complicated if some towers are shared by adjacent cells, as is usually the case. The shared towers carry extra weight but their moments are opposite in some degree, so their strength may be comparable to that of a tower serving only a single load. For sharing any fraction of the number of towers, the total number varies with height approximately as follows:

Number of towers $\propto 1/h$ to $1/h^2$ (47)

The greater exponent is the upper limit for no sharing of towers.

The total volume of all towers then has a variation within the limits:

Total tower volume $\propto$ cst to $h^3 \propto h^{3/2}$ (say) (48)

The <u>number of insulators</u> is more closely determined than the number of towers, because there is less sharing of insulators. The total number varies about as follows:

Number of insulators $\propto 1/h^{3/2}$ to $1/h^2$ (49)

Since the length of every insulator is determined mainly by the voltage (constant) and the total cross-sectional area mainly by the wire weight (also constant, 22):

Total volume of insulators $\propto$ cst (approx.) (50)

kgs

There are accessories which are external to the aerial structure but are closely related in requirements and cost. Such accessories include the ground system, the tuning reactors, and the heating power (for melting or preventing ice on the wires).

The ground system, in relation to the aerial system, determines the ratio of ground losses over radiated power. There are two kinds of ground losses that follow different rules, so will be considered separately.

The ground loss by magnetic field and radial currents requires radial wires in number and extent sufficient to hold this loss below a specified limit. Within the "height circle" (a radius equal to the effective height) the required number of radials and their length vary respectively with $1/h$ and h, so this length of buried wire is nearly constant. But a larger height circle leaves less area outside to be covered, so there is a small variation of total length of wire with height:

$$\text{Total length of wire} \propto 1/h^{1/2} \text{ (say)} \qquad (51)$$

The ground loss by electric field and vertical currents requires buried wires close enough to hold this loss below a specified limit. Increasing the height increases the radiation power factor and decreases the required number of wires over the area below the aerial:

$$\text{Total length of wire} \propto 1/h^2 \text{ (approx.)} \qquad (52)$$

Over most of the area, the radials (51) are sufficient to fill both needs, but the two losses are comparable and some compromise between these two rules is likely to be the best:

$$\text{Total length of wire} \propto 1/h \text{ (say)} \qquad (53)$$

The tuning reactors for the lower frequencies are only inductors. The air-core loading inductors and the iron-core keying inductors follow different rules.

kgs

The air-core reactors have a size determined mainly by the required ratio of reactance/resistance (Q or inverse power factor) and secondarily by the voltage. Because of eddy-current losses, the writer knows of no simple rule for variation of size with requirements. The size, or at least the volume of conductor, increases with Q and 1/h, so a nominal variation is assigned:

$$\text{Volume of conductor} \propto 1/h^2 \text{ (say)} \tag{54}$$

This is somewhat more than the variation of $1/h^{3/2}$ given by theory without reference to eddy-current losses and high voltage.

The iron-core reactors have a size determined mainly by the core volume and secondarily by the voltage and its required wire size. The core volume is proportional to the reactive power and hence the radiation Q of the antenna.

$$\text{Volume of iron core} \propto 1/h \tag{55}$$

The heating power is determined mainly by the surface area of the aerial wires, which is proportional to their total length (4) as given above (26). While this is only an occasional function, and may be used only on one half while its half of the transmitter is out of service, it may still represent a substantial amount of equipment and available power capacity.

The cost is determined by these various factors in a manner that evades exact formulation but indicates some simple trends. Some of the above relations have a direct influence on the cost, and these can be grouped in three classes.

Two factors contribute costs nearly invariant with height:

(22) Total weight of wire $\propto$ cst (approx.)
(50) Total volume of insulators $\propto$ cst (approx.)

The structural factors contribute costs that have a net increase with height:

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(5) (30) Wire separation $\propto h$ (approx.)
(14) Height of towers $\propto h$
(16) Lengths of wire spans $\propto h$
(47) Number of towers $\propto 1/h^{3/2}$ (say)
(48) Total tower volume $\propto h^{3/2}$ (say)

Taking these factors into account, the following relation seems reasonable for the total cost of towers, guy wires, supporting cables and mechanisms; it favors the total weight more than the number of towers:

$$\text{Cost of supporting structures} \propto h^{1/2} \tag{56}$$

The accessories contribute costs that vary inversely with height:

(26) Heating power $\propto 1/h$
(53) Total length of wire (in ground) $\propto 1/h$ (say)
(54) Volume of conductor (in tuning inductor) $\propto 1/h^2$ (say)
(55) Volume of iron core (in keying inductor) $\propto 1/h$

Expressing roughly the variation of the total of these costs:

$$\text{Cost of accessories} \propto 1/h \tag{57}$$

The height for <u>least cost</u> is that which establishes a certain ratio between increasing and decreasing costs, as follows:

$$\frac{\text{Cost of supporting structures}}{\text{Cost of accessories}} = 2 \tag{58}$$

The uncertainties in these estimates are such as to justify only the general rule that these two costs (56) and (57) should be comparable and preferably the former somewhat greater.

Therefore the shape variation with the specified invariant performance leads to this rule for least cost. This conclusion is reasonable and the preceding discussion may help in perceiving the wide diversity of individual contributing influences.

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Relations of Performance, Size and Cost.

Assuming the shape that gives some specified performance with least cost, there will be given some relations of performance, size and cost, for an aerial structure of this shape.

It is assumed that the shape for least cost remains the same for the useful range of sizes, and this shape is assumed. All dimensions vary in the same ratio except for the wire size, which is held constant for reasons to be given. (The details of the insulators are not included in the shape.) The size is much smaller than the radian hemisphere, so the aerial behaves as a pure condenser.

With the wire radius (a) remaining constant, the power limits imposed by voltage and gradient vary in nearly the same ratio over the useful range of sizes. It is assumed that these two limits are the same (the balanced design of p. 323-6, esp. (8) on p. 324). Also the constant wire size retains a constant ratio of excess gradient for corona (p. 307-8).

Since the shape is constant, the variation of size may be expressed by any one dimension; the effective height (h) is here taken as the index of size.

Each relation between two variables, with all other variables constant, will be expressed in the form of a product or quotient that is invariant (constant). For example, the following constants are shape factors independent of size:

$$A/h^2 = \text{cst (spreading ratio)} \quad (1)$$
$$A_a/h = \text{cst}; \quad l_a/h = \text{cst} \quad (2)$$
$$C/h = \text{cst (approx.)} \quad (3)$$

in which

h = effective height (index of size)

A = effective area

$A_a = 2\pi a l_a$ = conductor area

l_a = length of wire

a = radius of wire (constant)

C = capacitance

kgs

The relation for capacitance is an approximation which ignores the minor change of shape by leaving the wire radius constant.

A. Relations between performance and size at constant frequency.

In this section, the wavelength (λ) is constant and other limitations will be stated. The other parameters are the size (h) and the following:

P = radiated power
V = antenna voltage (limited by insulators)
E_a = wire gradient (limited by corona) which is related to V so need not be stated.

The radiation power factor (ρ), the bandwidth and the speed of keying are here omitted, since not now regarded as the principal limitations.

Constant V; p. 323(4): $P/h^4 = cst$ (4)

Constant P; p. 323(4): $Vh^2 = cst$ (5)

Since there is an upper limit on the gradient on the wires and on the voltage on the insulators, any variation of voltage is an opposite variation of the margin between operating conditions and the upper limit.

B. Relation between size and cost.

With the wire radius (but not its density) held constant, and the shape otherwise invariant, the previous discussion of a single cell gives some indication of the variation of properties of various parts of the structure (p. 331-40). It appears that the volume and weight of material in the towers (including guy wires and supporting cables) is roughly proportional to h^3 while the weight of wire is nearly constant. If the cost varies less than the weight of the towers, it is reasonable that the cost of the structure might be proportional to h^2:

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$$S/h^2 = \text{cst (say)} \tag{6}$$

in which

S = cost of aerial system

This rule will be assumed in stating some other relations involving cost.

C. Relations between performance and cost.

The following are obtained by combining some of the above relations.

Constant V: $$P/S^2 = \text{cst} \tag{7}$$

Constant P: $$VS = \text{cst} \tag{8}$$

The latter relation shows how the cost can be decreased by working closer to the upper limit of voltage.

One test of these relations is interesting for its simplicity. If the shape is not critical, one way of doubling the cost (S) is to substitute two structure for one, and separated just enough to have negligible interaction. These two relations are valid for this case. (?)

One basis for comparing performance is the relative bandwidth and speed of code communication to a receiver whose sensitivity is limited by thermal noise. Four times the power and two times the cost enable four times the bandwidth and speed. Here it is assumed that frequency-shift keying (FSK) is used in such a manner that the bandwidth of the transmitting antenna is not a limitation.

Another basis for comparing performance is the relative depth of submersion of a submarine receiving antenna to intercept signals of equal strength. At 15 Kc, four times the power and two times the cost enable the depth to be increased by 1.4 meter or 4.5 feet.

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D. Relations between performance and wavelength.

If the wavelength (λ) varies with invariant size and cost, the other quantities fit into the following pattern.

Constant V: $P\lambda^4 = cst$ (9)

Constant P: $V/\lambda^2 = cst$ (10)

Note the comparison between these and (4), (5).

E. Relations between wavelength and cost.

The size is increased with wavelength to retain the same performance:

$h/\lambda = cst$ (11)

$S/\lambda^2 = cst$ (12)

kgs

Wire in Shield for Corona Test.

There is a need for a corona test of a section of wire under controlled conditions approximating the operating conditions in the vicinity of the aerial wires. An arrangement is here proposed which meets these requirements. A length of wire (of the kind to be used in the aerial) is mounted horizontally in a shield such that the gradient on the wire can be computed from the dimensions and the applied voltage. At a frequency near the lowest operating frequency (14 Kc) the voltage on the wire is increased until corona is detected. The mounting is designed to avoid corona at other places. A spray over the wire provides wet conditions. (Ice and snow conditions should also be tested.)

The diagram shows a horizontal wire covered by a shield tent. These approximate a section of coaxial line susceptible of simple computation.

The essential dimensions are shown on the cross-section. The gradient on the wire is

$$E_1 = V/d = \frac{V}{r_1 \ln r_3/r_1} \qquad (1)$$

in which

E_1 = voltage gradient on wire (RMS volts/meter)
V = voltage on wire (RMS volts)
d = V/E_1 = effective distance (meters)
r_1 = radius of wire (meters)
r_3 = 1.134 h = effective radius of shield (meters)

The convenient rating of the structure is the effective distance,

$$d = r_1 \ln r_3/r_1 \qquad (2)$$

The shield is a cylinder whose cross-section is an equilateral triangle. In its relation to the wire, it behaves as a circular cylinder of radius equal to 1.134 times the distance (h) between wire center and each side.

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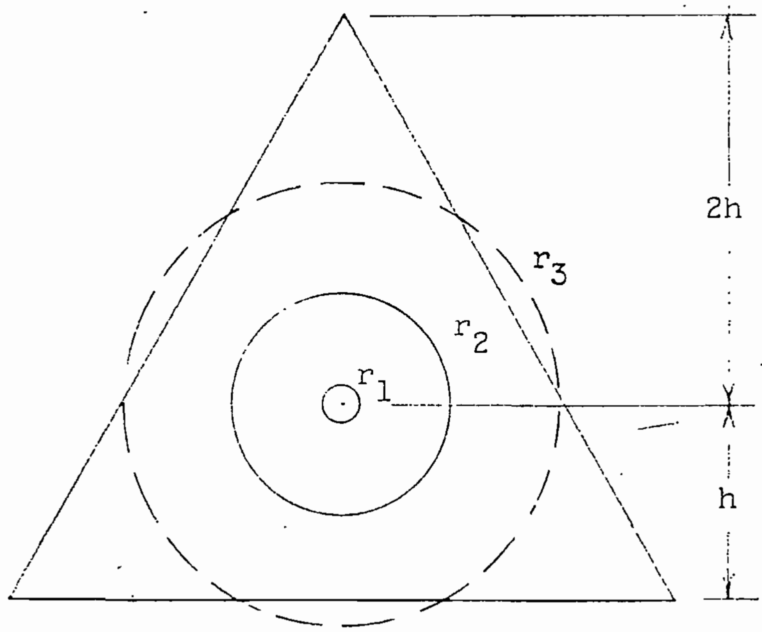
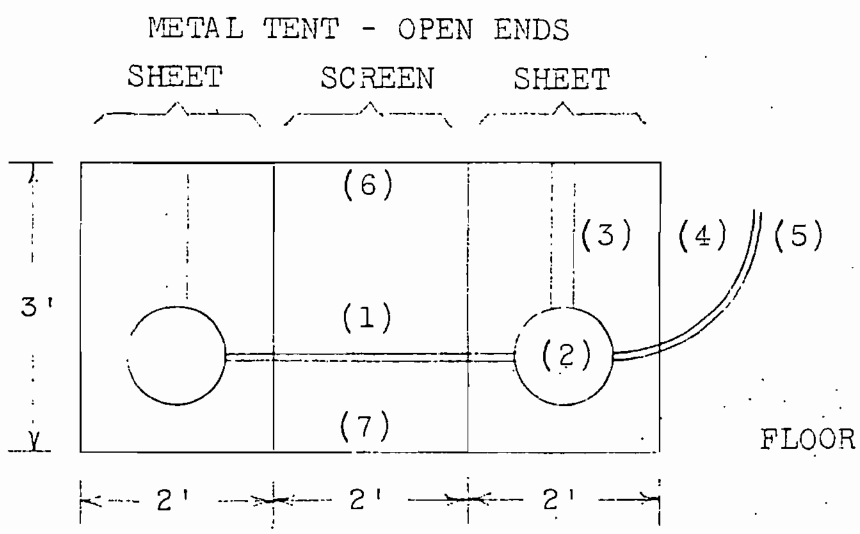


Cross-section



Side view

Horizontal wire in shield tent.

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The ends of the wire are mounted in corona shields to avoid excess gradient. A sphere of radius r_2 inside another sphere of radius r_3 (roughly approximated by the tent) has a gradient

$$E_2 = \frac{V}{r_2(1-r_2/r_3)} \tag{3}$$

If the outer radius is specified, this gradient is minimized by making the inner radius 1/2 as great.

$$E_2 = 4V/r_3 \tag{4}$$

The ratio of gradients on corona shield and wire is

$$E_2/E_1 = \frac{4r_1}{r_3} \ln r_3/r_1 \tag{5}$$

If $r_3/r_1 = 24$, $E_2/E_1 = 0.53$.
This is taken as a reasonable minimum size, leaving on the spheres a gradient about 1/2 of that on the wire.

The following dimensions are proposed; it is noted that the size of the sphere is noncritical, so a convenient value is specified.

r_1 = 0.5" = 12.7 mm
r_2 = 6"
h = 12" = 305 mm
r_3 = 1.134 h = 346 mm
r_3/r_1 = 27.2
d = 42.0 mm

This last quantity relates the voltage and the gradient. For example, 1 Kv/mm gradient on the wire surface requires 42 Kv between wire and shield.

The side view shows the structural arrangement. The tent has a sheet-metal floor and each wall is made of three panels; the end panels are of sheet metal to avoid excess gradient, while each center panel (where the gradient is much less) is made of wire-mesh screen for visual observation. Telescope and camera may be provided at one side, at the height of the wire. The room should be darkened for optical observation, but not necessarily for radio detection. The ends of the tent are open, with edges rolled back or otherwise rounded to avoid corona. The following notes are keyed to the diagram.

(1) The 3-ft. length of wire has substantially uniform field along the middle one foot of length for observation, and lesser field toward the ends. The stranded aerial wire should be tested, and also smooth wire (or tubing) for comparison.

(2) The corona shield at each end is smooth and approximately a sphere of 1 ft. diameter. A cylinder with rounded edges may be used.

(3) The hanging support at each end is an insulating rod; porcelain is recommended. The centering of the inner conductor is noncritical.

(4) The open ends of the tent may have rolled edges or round metal tubing on the outside as borders (radius at least 1").

(5) The connection to the high-voltage generator should be made of metal tubing somewhat larger than the inside wire, and kept at a greater distance from grounded objects. A diameter of 1.5" and distance of 2 ft. (except from floor and tent) are suggested.

(6) The water spray may be provided from a perforated metal pipe mounted just inside the tent at the top, where the gradient is greatly reduced by the acute angle. The spray should be directed on the wire, with an attempt to avoid the spheres.

(7) A radio probe may be inserted from one side of the tent near the floor and midway between the ends. It should be connected to a radio receiver through a coaxial cable grounded to the tent floor. Electric-field pickup is required for most sensitive detection of corona.

NB 83, p. 47-50.

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Effects of Leakage in Insulator String.

Conductive leakage across any of the insulators in a string causes an increase of voltage on the other units. Such effects have been estimated for an idealized case, described as follows:

Number of units in string	10
Voltage distribution in string (normal)	uniform
Total voltage on string	180 Kv RMS
Voltage on each unit (normal)	18 Kv RMS
Operating frequency (VLF)	14 Kc
Space capacitance at each junction	13 µµf
Direct capacitance in each unit	55 µµf
Direct susceptance in each unit	4.8 µ-mho
Corresponding reactance	0.21 M-ohm
Corresponding capacitive current	86 ma RMS
Direct leakage conductance (where specified)	2.4 µ-mho
Corresponding resistance	0.42 M-ohm
Corresponding conductive current	43 ma RMS
Corresponding power loss (per unit)	0.78 Kw
Voltage ratio of attenuation (per unit or section)	0.63

This description is intended to approximate the behavior of a string of 12 units, Lapp No. 43709, graded for less than 1/10 the total voltage on any one unit. The amount of leakage is taken to give conductive current half as great as the capacitive current, this amount to be specified for some part of the discussion. It is conceivable that this amount of leakage might occur during a transient combination of ice and water on the porcelain cone.

The 14-Kc operation is radically different from 60-cycle operation, in that the direct capacitive current (86 ma) predominates over any reasonable amount of conductive current. This is in contrast with .037 ma at 60 cycles, which would often be exceeded by conductive current. Therefore, under adverse conditions, the voltage distribution at 60 cycles is determined mainly by the unstable conductive currents while that at 14 Kc is determined mainly by the stable capacitive currents. The effect of conductive currents is so much

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less at 14 Kc that the precautionary rules of 60-cycle experience are likely to be inapplicable.

The worst condition is to have leakage across all except one of the units in the string. Then the remaining unit takes more than its share of the voltage. At 60 cycles, it would take nearly the entire voltage. An approximate computation for 14 Kc indicates that this extreme condition would increase the voltage on the one unit by the factor 1.19, to 21.4 Kv. This is only a small increase, and the probability of its occurrence is extremely small.

If this amount of leakage occurs across alternate units in the string, the remaining units would have their voltage increased by the factor 1.10, to 19.8 Kv. This distribution of leakage is representative of more probable conditions, although the assumed amount of leakage still has a small probability of occurrence.

For comparison, let us consider the effect of a short-circuit on one unit near the middle of the string. Each of the two adjacent units has its voltage increased by the factor 1.19, to 21.4 Kv. This amount is comparable with the worst leakage effect described above.

NB 84, p. 6, 25-8.

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Ice on Wires.

Ice on the aerial wires greatly increases the losses, in the same manner as frozen ground around the buried wires. Reference is made to the formulas on p. 423-4, especially (4) with substitution of ln a'/a (6).

When icing starts, one half of the antenna is connected to the de-icing circuit while the other half continues to operate for 1/2 hour, then vice versa for the next 1/2 hour, and so on.

It is expected that the ice will not exceed 1/2" radial depth. The following example is computed for this depth on wires of 1" diameter, so the ice doubles the diameter (a'/a = 2). The length of wire (l) is the total in the operating half of the aerial (including downleads and cross-cables).

$$f = 14 \text{ Kc}$$
$$\lambda = 21.4 \text{ Km}$$
$$h = 150 \text{ Km}$$
$$l = 50 \text{ Km}$$
$$P/P_o = 18/k$$

The effective dielectric constant (k) is assumed to be accompanied by effective shunt conductance to give a phase angle of about 45°. This determines the power ratio (P/P_o) of loss over radiation. The following table gives this ratio for various conditions of dielectric in the space of 1/2" radial ice. (See p. 425-6.)

Dielectric	k	P/P_o
Worst that can be imagined	1	18
Worst condition of dense ice	3	6
Fresh snow (NEL)	4	4.5
Packed snow (NEL)	9	2

The last condition is regarded as the most likely approximation of ice forming near the freezing temperature. If the half-antenna is operating at its rated voltage and current, the radiation is 0.25 Mw and the loss in 1/2" ice is 0.5 Mw. This extra power is available from the transmitter, because this and other losses (say 0.25 Mw) bring the total to 1 Mw normally delivered to each half. There remains a difficult problem of continuously adjusting the coupling to maintain impedance matching between transmitter and antenna while the ice is increasing.

NB 83, p. 114-5.

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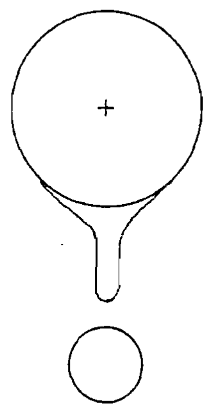
Water Drops and Precipitation.



Water drops from an aerial wire are the cause of corona at voltages much too low to cause corona on the clean wire. The direct cause of corona is the excess electric gradient on the lowest side of the falling drop, which is several times that on the clean wire.

The diagram shows the formation of a water drop under a wire, approximately actual size. (See Edgerton, "Flash", 1939, p. 125.) The drop forms a long stem which has just broken off at the instant shown. This occurs when the drop is separated from the wire by a distance about twice the diameter of the drop.

A test of water dripping from a faucet has yielded the size of a spherical drop. It is larger than might be expected.

Diameter of spherical water drop:	0.34 inch = 8.6 mm
Volume " " " " :	330 mm^3
Number of drops:	$3 \times 10^6/meter^3$

It is estimated that a drop has experienced free fall for a distance equal to about twice its diameter when the stem breaks.

Distance of free fall:	16 mm
Time " " " :	.057 sec
Speed at end of this time:	0.56 m/sec

The water is a fair conductor so the falling drop is at the potential of the wire until the stem breaks. Then there may be a small spark in the gap at the break, which would hold the connection a

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little longer. The stem quickly collapses into a much smaller droplet, whereupon the spark would probably stop.

The acceleration of the falling drop is caused partly by gravity and partly by electric repulsion. The following example is typical of conditions under a wire of diameter one inch.

Gradient on wire surface:	0.7 Kv/mm (RMS)
Gradient at water drop (if disconnected) in middle of free fall, with center 12 mm below the wire:	0.35 Kv/mm (RMS)
Voltage difference between wire and water drop (if disconnected):	6 Kv (RMS)
Capacitance of water drop:	4.4 μμf
Charge on water drop at this voltage difference:	.026 μ-coulomb
Electric force on this charge:	.009 newton
Acceleration of gravity:	9.8 m/sec^2
Mass of water drop:	.00033 Kg
Gravity force on this mass:	.0033 newton
Ratio of electric/gravity forces:	2.7

The electric "voltage difference" with the water drop "disconnected" is used to compute the charge that is induced on the water drop when it is connected to the wire. This charge is accelerated by the gradient in this location, assuming that the charge on the wire is little affected by the proximity and size of the water drop.

From the foregoing example, it is concluded that the total downward force may be about 4 times that of gravity. This would modify some quantities as follows:

Time of accelerated fall:	.028 sec
Speed at end of this time:	1.1 m/sec

These values will be assumed in the following.

The electric force on the water drop discontinues when the drop is disconnected from the wire. This occurs after the stem breaks and subsequent sparking stops. However, the subsequent course of the water drop has no appreciable effect on the antenna behavior.

There may be sparks also in the gaps between the wire and raindrops falling onto the wire. Since these drops are caused by condensation, they may be of any size. They provide the wire with the water coating which disintegrates into falling drops.

The standard precipitation (ASA-AIEE) is very heavy rain, probably the maximum commonly experienced:

Standard precipitation: 0.2 inch/minute = .08 mm/sec

An aerial is assumed, with the following conclusions relating to rainfall.

Length of wire:	100 Km
Diameter of wire:	25 mm
Interception area:	2500 m^2
Precipitation intercepted:	0.2 m^3/sec
Water dripping off:	600,000 drops/sec
Average number of falling drops at any instant:	600,000 x .028 = 17,000 drops
Average length of wire per falling drop:	6.0 m

This gives some idea of the number of falling drops that may be causing corona at any instant.

A cross-wind would probably blow off the drops before they develop the size described above. This might increase the maximum gradient but on a smaller radius. It is conjectured that these two factors together would leave the corona power loss about the same.

NB 83, p. 117-50; NB 84, p. 5, 8-9.

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Excess Gradient on Stranded Wire.

A stranded wire or cable has a maximum gradient greater than that on the equivalent smooth round wire. For evaluation of the excess gradient, it is assumed that the outer surface of the stranded wire is made of many round wires in contact, so each strand is much smaller than the composite wire.

A method of approximation has been devised which leads to ratios in terms of the following quantities:

a = actual radius of smooth wire
a = equivalent radius of stranded wire
$a + \Delta a$ = outer radius of stranded wire
c = strand radius
E_a = constant gradient on smooth wire
E_c = maximum gradient on stranded wire

The equivalent radius of a stranded wire is the radius of a smooth wire having the same capacitance.

The ratio of excess gradient on the stranded wire is

$$E_c/E_a = 1.39 \pm .01$$

This ratio would indicate a corona voltage lower than that for a smooth wire, in the ratio $1/1.39 = 0.72$. However, the start of corona is influenced by the average gradient over a limited radial distance. Tests on typical high-voltage power lines indicate that the 60-cycle corona voltage on clean stranded wire is lower in a ratio of about 0.92 .

The effective radius of a stranded wire is less than its outer radius by a fraction of the strand radius:

$$\Delta a/c = 0.113 \pm .002$$

This difference is usually negligible.

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The above approximate values may be compared with exact values for the extreme case of 2 strands in contact:

2 strands (exact)	many strands (approx.)
$a + \Delta a = (4/\pi)a = 2c$	$a + \Delta a >> 2\ c$
$\Delta a/c = 8/\pi - 2 = 0.545$	$0.113 \pm .002$
$E_c/E_a = \pi^2/8 = 1.235$	$1.39 \pm .01$

The limit of many strands is approached rapidly with increasing number so the residual error is much less than the difference in this comparison.

NB 84, p. 13-24.

See Langton MONO

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To Developmental Engineering Corp.

VLF ANTENNA NOTEBOOK - GROUND SYSTEM

This is the fourth of a series of reports for collecting information relating to the design of VLF antennas, especially a high-power antenna for 15 Kc. Each report will be cumulative and will be assigned a block of numbers so that any page in this series will be identified by its page number. The report numbers are being assigned from the block of 301 to 309. The last digit of the report number will be the first digit of the 3-digit page number.

Contents   Page

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Comparison of Central Grounding and Multiple Grounding.

Multiple grounding is the principal objective of the usual multiple tuning of an antenna that is spread out over a large area (but not as large as the radiancircle).

The principal justification for the complication of multiple grounding is the idea that the resistance of a single ground connection could not be reduced below a certain value (of the order of one ohm). This is true in some philosophies of grounding, but not in general. The use of radial wires enables the resistance of a ground to be reduced indefinitely by increasing the number and length of these wires.

The choice between multiple grounding and central grounding is then determined by which one offers the preferable combination of features relating to performance, cost and operation. The following outline gives some of the relative advantages of either, for a specific case.

The antenna arrangement chosen for this comparison is a pair of long aerials side-by-side, together making the complete antenna. These two halves are separated enough so that either can be operated alone while the other is out of service. Each half is complete with tuners, power amplifiers and accessories. The transmitter house is located in the middle between the two halves. If a central ground is used, it is here. If multiple grounds are used, one is located toward each end of each half, and 2 are located near the transmitter house, making 6 in all; the last 2 might be combined into one.

As will appear in the outline below, the simplicity and other advantages of central grounding impose a heavy burden of proof before choosing multiple grounding. In the present state of the art, either system is capable of reducing the ground losses below any reasonable amount without prohibitive cost. The writer is inclined to favor central grounding as offering the preferable combination of features. When all factors are considered, it is conjectured that the same electrical performance could be obtained at the same cost, leaving as a net advantage the great operational benefits of a centralized installation.

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Advantages of Central Grounding (C)	Advantages of Multiple Grounding (M)
C-1. Simplicity and economy of central housing of tuners, controls, personnel for manual operations; reliability of communications and coordination; accessibility for transportation; avoidance of lines for intercommunication.	
C-2. A central tuner, for the same cost, has substantially less losses, since larger coil has lower power factor (higher Q).	
C-3. De-icing facilities centralized are more efficient and simpler; personnel are at hand if manual switching is needed.	
C-4. The inductance of the central downleads, carrying more current, is sufficient for partial tuning of antenna at lowest frequency, reducing the requirements on tuning coils and lead-in insulators (say, to 1/2 inductance and 1/2 voltage).	M-4. A greater ratio of tuning is possible because the multiple downleads present less inductance.
C-5. Lead-in insulators reduced to 2 at 1/2 voltage instead of 6 at full voltage.	
C-6. Keying inductor (if saturable-reactor type is used) can be designed in single unit for entire current of each half of antenna, which enables	

lower voltage and lesser losses (because of larger unit and lower-voltage insulation); the keying circuits require no remote wiring; if cooling is needed, the central type requires only a single cooling equipment.

C-7. Experience of most recent installation, Jim Creek, the only one having two halves for independent operation.

M-7. Experience of older installations, such as German Goliath and Annapolis.

C-8. One-point grounding system has simplest pattern of radial wires, though current is concentrated in some degree below the 2 downleads and in lesser degree below the 2 antennas.

M-8. The same total number of radials requires lesser total length of wire and gives lesser average distance for current in the wires, so ground losses would be somewhat less.

C-9. Near lead-in, the electric-field ground losses are reduced by 2 downleads instead of 6, and further reduced by closer spacing of radials underneath.

C-10. Fewer downleads (2 instead of 6) cause less load on aerial supports and less submarginal capacitance to ground.

M-10. The average length of 4 vertical and 2 oblique downleads is less than that of just 2 oblique downleads, so current losses are less for same amount of wire.

M-11. The average distance for current in the aerial wires is less and the mean-square current is less so the losses are much less (though small in either case).

Principles of Ground System.

The ground system in the vicinity of a grounded vertical antenna is the combination of conductive ground and installed conductors, which together carry the base current of the antenna and the ground currents of the radiated wave. It usually includes a pattern of conductors above the surface or buried a little below the surface, such as radial wires. They serve to carry most of the radial current, as far as they extend, and to serve as a connection to the conductive ground in the region beyond.

The ground currents from a vertical radiator are radial. If the radiator is a vertical conductor 1/4 wavelength in height (one radianlength in effective height) over a perfectly conducting surface, the total radial current is constant (independent of radial distance). If the effective height is much less than one radianlength (as in a VLF antenna) the total radial current is much greater at distances much less than one radianlength. Therefore the most severe requirements for ground conduction are within the radiancircle. The excess current is the quadrature component associated with the magnetic and electric energy stored in the volume of the radian hemisphere.

Concentration of currents. In a practical system, there is some concentration of currents under any aerial conductors carrying large currents, since the ground currents are boundaries for the magnetic field of these aerial currents. Such a concentration occurs below oblique downleads and horizontal aerial wires, disturbing the radial pattern.

The phase angle of ground currents is nearly constant from the base out to a distance of one radianlength, reflecting the nature of these currents as currents in the aerial condenser (as distinguished from the radiation currents that are retarded in phase angle beyond this distance). Because of this constancy of phase angle, any ground resistance in the radiancircle is reflected as excess resistance in the antenna circuit (whereas ground resistance at some greater distances is reflected mainly as a change of reactance or even as a negative change of resistance).

Ground losses by radial currents are the losses usually considered. They are proportional to the series-resistance component

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of the ground impedance, this being mainly inductive reactance in the region of closely spaced radial wires. These losses can be reduced indefinitely by increasing the number and length of radial wires.

<u>Ground losses by vertical currents</u> are usually ignored. This is a "dielectric loss" in the sense that the vertical currents are capacitive currents between the aerial wires and the conducting layers or wires in the ground. These losses can be reduced indefinitely by locating the radial wires above the surface, or can be minimized by burying the wires at the optimum depth. In areas where ground conduction in the skin depth (without wires) is relied on, these losses may be negligible in comparison with the radial-current losses.

<u>The radial reactance</u> of ground wires is determined mainly by the number of wires, being inversely proportional to the number. The reactance in itself is harmless, but it causes a radial voltage gradient which in turn causes radial currents in the ground. In other words, the radial reactance is a measure of the leakage of magnetic field into the ground, in spite of the shielding effect of the wires.

<u>The skin depth</u> in the ground is defined in the same manner as the skin depth in metallic conductors, but its value is much greater. The radial currents under the radial wires flow mainly in the region between the surface and the skin depth. In accordance with the behavior of the skin effect, the ground currents in the skin depth present equal values of resistance and reactance; these may be evaluated as series or parallel components.

<u>The radial resistance</u> of the ground system is effectively the series-resistance component of the impedance of the radial wires and the ground conduction in parallel. It is this resistance and the radial current that determine the ground losses usually considered.

<u>The maximum radial resistance,</u> for any pattern of radial wires, is caused by a certain relation between the radial reactance and the ground conduction (in the skin depth). The maximum possible value of radial resistance is about 1/5 the radial reactance of the wires. This relation makes it possible to hold the radial resistance less than any specified value for all conditions of the ground underneath, so the ground properties need not be relied on. In usual conditions, the ground properties are likely to be of the order of the

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worst values.

Designing for the worst ground is a philosophy that is enabled by the judicious use of radial wires. It is more reliable than any assumed limits of ground conditions. It is economical in obviating the need for complete tests and information on ground properties. It may be slightly wasteful in specifying radial wires in an amount greater than the minimum required, but with some improvement in efficiency.

The sphere of influence of a "small" antenna is logically defined as the radiansphere, since this is the space occupied mainly by the "stored" energy of a resonant antenna (as distinguished from the radiated power). For a grounded antenna, this space becomes the radian hemisphere, bounded on the ground plane by the radiancircle. Within this circle, the radial currents are increased greatly by the capacitive currents of the resonant antenna. It is logical to regard the ground losses in this circle as a property of the antenna, and to regard further ground losses as a property of the propagation path.

Length of radial wires. Since the ground currents of a "small" antenna are much larger within a distance of one radianlength, it is natural to consider radial wires out to that distance. The required length may be evaluated more closely by considering also some other factors.

Salt water of the ocean is an excellent ground conductor for a VLF antenna. Except for the region of high current at the base of the antenna, it is adequate for all ordinary needs; it is likely to be better than any reasonable number of radial wires. Some antennas are located on a salt marsh to obtain ground conditions nearly as good as salt water.

Terminating radial wires. Beyond the ends of radial wires, the ground conduction must be relied on. The transition between wire conduction and ground conduction may occur by a gradual transfer of current from the wires to the ground, over a distance comparable with the skin depth in the ground. If the ground is dry near the surface, where the wires are buried, and wet at a convenient depth below the surface, there is an advantage in connecting the ends of the radial wires to metal ground rods driven into the wet ground (to a depth comparable with the skin depth in the wet ground). A transition between radial wires and salt water may be made either way, by extending the wires into the water or by connecting the ends of the wires to ground rods near the water (on a sandy beach, for example).

Best utilization of radial wires is obtained by distributing the wire so that the current in a wire is about the same at all locations in the pattern. The principal objective is to reduce the radial-current losses below a specified amount with the least total length of radial wires, which requires the best utilization of the wire.

Binary zoning is one pattern of radial wires that is simple to plan and install. The radius is marked at binary fractions (1/2, 1/4, 1/8, etc.) of the radianlength, and the number of radial wires is increased in binary ratios (2, 4, 8, etc.) in going from each zone to the next smaller zone. The smallest zone or inside circle should have a radius comparable with the effective height of the antenna, within which the radial current is nearly constant. At each circle between adjacent zones, all radial wires are joined to a circle of wire. If radial wires are extended beyond one radianlength, the number of wires may be 1/2 the number in the outer zone, and should not be further reduced.

The radial wires may be made of any material having a conducting surface that meets several requirements. For conduction, the surface should be nonmagnetic metal to a depth of at least a fraction of the skin depth in this metal. For contact with the ground, the surface area is a requisite. For endurance, the surface coating must resist corrosion, and the cross-section must be strong enough to hold in prevailing conditions of weather and surface loading (by working machinery). A metal strip (say 2" wide) meets these needs with economy of material; also it offers flexibility for reeling and ease of bonding at junctions. The inductance of a thin strip is equal to that of a round wire whose diameter is 1/2 the width of the strip; ~~also~~ the same rule applies to the effective surface area.

Buried wires are preferable to overhead wires for maintenance and meet all ordinary requirements (perhaps excepting a small area at the base of the antenna). There is an optimum depth for minimizing ground losses. Lowering the wires increases the radial-current losses in ground conduction, but only slightly if their depth is much less than the skin depth. The vertical-current losses are minimized by lowering the wires to a depth greater than the wire diameter but less than the separation of adjacent wires. Considering these factors, a depth of about one foot is near the optimum for reducing losses and for practical considerations of installation and maintenance.

The hub of the radial wires is designed to accomodate the required number of wires. A large number of wires (say 100 or more) may be bonded to a large metal sheet used for the base connection. The connection may be made to the sheet by one or more stranded cables; it is sufficient to use the kind of cable used in the tuning coil. This metal sheet might form the base of the entire housing of the tuning coil; it may have openings over a fraction of its area.

Metal structures for supporting the aerial wires are likely to carry to ground a small but appreciable fraction of the total antenna current. Therefore metal towers and uninsulated guy wires should be tied into the pattern of ground wires. For example, 4 radial wires from the base of a tower may be bonded to the nearest 8 of the main radial wires.

Design formulas will be given for the pattern of radial wires that is required for a central grounding system to hold the ground losses below any specified amount. The problem may be complicated by multiple grounding and by long feed lines with ground return.

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Symbols.

Supplement to p. 103, with some specialization for ground system.

Radiation quantities (sub-o):

P_o = $R_o I_o^2$ = power radiated
R_o = radiation resistance
I_o = antenna current (total, center, base, r = 0)
h_o = effective height
r_o = effective radius of disc (flat top)

Various electrical quantities:

P = power dissipated (losses)
R = dissipation resistance
R_w = surface resistance of square area, wired
G = conductance
B = susceptance
Z = R + jX = impedance
Y = G + jB = admittance
I = total radial current at any radius (r)
I_1 = total radial current of radiation term (constant) I_c
I_1 = 158 amp if P = 1 Mw I_c
σ = conductivity of ground (land or sea)

Space quantities:

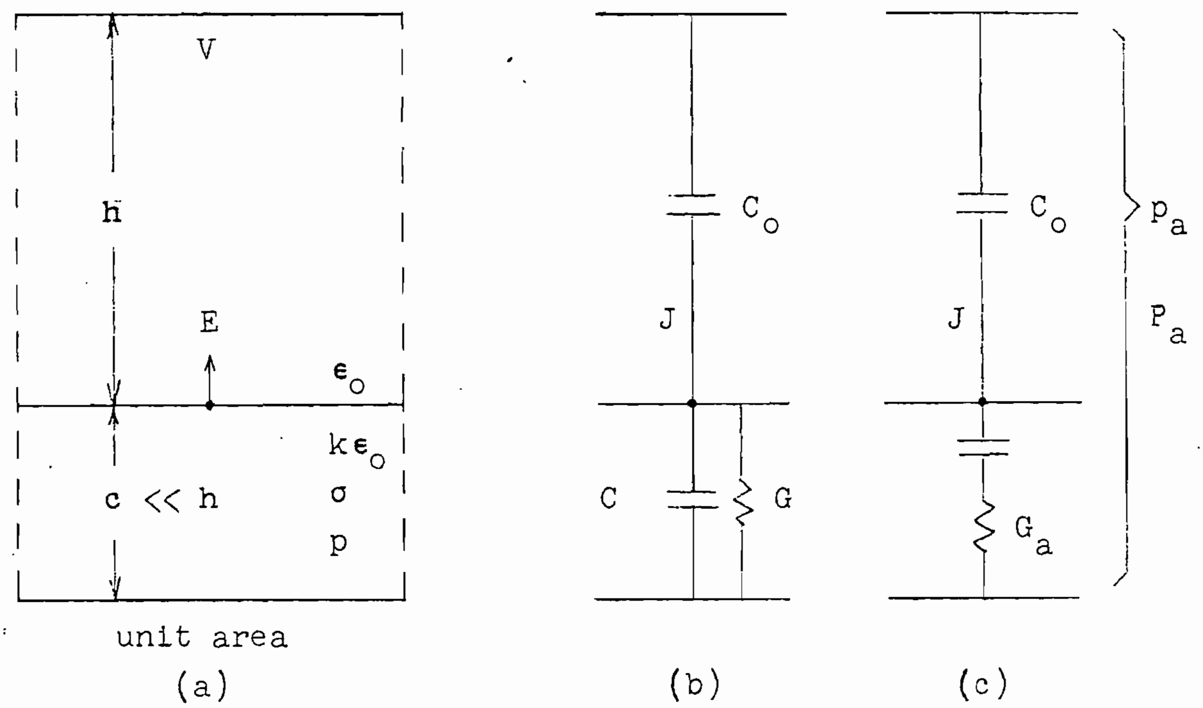
J = current dens. NB p.75
G_a
P_a

a = radius of buried wire
a' = effective radius of sheath around wire
b = depth of buried wire
c = depth of equivalent plane sheet
d = pitch of buried wires $1/d$ = density of wires
l_a = total length of buried wire
A_a = total area of surface of buried wire
N = number of radial wires in circle

NB 83, p. 54-5.

$2\pi r/\lambda$ = distance angle $2\pi h/\lambda$

Area Losses by Vertical Electric Field.



Unit area of ground with plane sheet conductor buried under surface.

The vertical E field is accompanied by capacitive currents which accumulate to form the radial currents associated with the horizontal H field. In an area filled with buried wires to carry the radial currents, the vertical currents must be carried from the surface to the buried wires through the ground, causing losses that are identified with the E field. (See p. 406.)

These losses have usually been ignored in previous studies, but are found to exceed the H-field losses under certain conditions. Some conditions that increase the E-field losses are the following:

Freezing ground.
Drying ground.
Covering with snow.
Covering with vegetation.

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The ground conditions are under the surface and therefore their effects depend on the field pattern around the buried wires. The coverings are above the surface and therefore are subjected to nearly uniform field.

The E-field losses will first be formulated on the assumption of a plane conducting sheet buried at a constant depth (c) below the surface, the space therebetween being filled with soil having the properties of a lossy dielectric of known dielectric constant (k) and conductivity (δ). This gives uniform field in the ground. Further on, there will be formulated an equivalence between this plane and the actual surface of the buried wires. The depth of the equivalent plane is usually comparable with the separation of the buried wires.

As an introduction, there will be formulated an idealized case based on a plane sheet aerial having a uniform vertical electric field (E) under its entire area, and none outside this area. Then the dissipation factor of the entire area is that of any unit of area, shown in the diagram. From the dissipation factor (p) of the ground, it is possible to compute in simple terms the dissipation factor (p_a) of the entire condenser.

The dissipation factor (or loss tangent) of the ground dielectric is

$$p = \frac{\sigma}{k\epsilon_o\omega} = \frac{\lambda\sigma R_c}{2\pi k} = \frac{60\sigma\lambda}{k} = \frac{\lambda^2}{2\pi^2 k\delta^2} \qquad (1)$$

in which

k = dielectric constant of the ground
σ = conductivity of the ground (mhos/meter)
δ = skin depth in the ground (meters)

This is equal to the power factor if much less than unity. In the ground, however, it may be much greater than unity.

In terms of the circuit parameters shown, the dissipation factor in the ground is

$$p = G/C\omega \qquad (2)$$

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The parallel C and G of the ground may be reduced to series components; the series conductance becomes

$$G_a = \omega C \frac{1+p^2}{p} \qquad \text{mhos/m}^2 \qquad (3)$$

The resulting dissipation factor of the entire condenser (since $C \gg C_o$) is

$$p_a = \omega C_o / G_a \qquad (4)$$

The ratio of capacitance above the surface to that below is

$$C_o/C = c/kh \ll 1 \qquad (5)$$

The dissipation factor of the entire condenser may be expressed as follows:

$$p_a = \frac{C_o}{C} \frac{p}{1+p^2} = \frac{c}{kh} \frac{p}{1+p^2} \qquad (6)$$

This is a simple form, entirely in terms of ratios.

If the conductivity of the ground is subject to variation over a wide range (as by freezing) so that its dissipation factor (p) goes through unity, the maximum dissipation factor of the condenser is

$$\max p_a = c/2kh \qquad (7)$$

This may be divided by the radiation power factor (p. 306) to estimate the ratio of power loss over power radiated:

$$\frac{\max p_a}{p_o} = \frac{3\pi}{2k} \frac{c(\lambda/2\pi)^3}{Ah^2} = \frac{3}{16\pi^2} \frac{c\lambda^3}{kAh^2} \qquad (8)$$

This relation may be expressed as a value of the equivalent depth, which must not be exceeded:

$$c = \frac{16\pi^2}{3} \frac{kAh^2}{\lambda^3} \frac{\max p_a}{p_o} \qquad (9)$$

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In a "small antenna", this indicates that c may have to be much smaller than the principal dimensions of the aerial.

In practice, the maximum value of E-field dissipation factor is reduced by such causes as the following:

Concentration of electric flux near the aerial wires.
Spreading of the electric flux over a larger area on the ground.
Termination of a fraction of the electric flux on grounded towers and guy wires bonded to the buried conductors.

In view of these complications, the practical problem may require a map of the electric field on the ground and an evaluation of the associated losses by partial areas, for which the rules will now be described.

The E-field losses may be computed on an area basis from the following two quantities properly defined for any specified area.

J = area density of vertical current (amperes/meter^2)
G_a = area conductance between surface and buried conductor (mhos/meter^2)

The density of the vertical dielectric currents is

$$J = \omega\epsilon_o E = \frac{2\pi E}{\lambda R_c} = \frac{E}{60\lambda} \qquad \text{amp/m}^2 \qquad (10)$$

The area conductance (G_a) is the reciprocal of the series-resistance component of the impedance of a unit area (1 m^2) between surface and plane conductor. It is preferred to express it in terms of conductance instead of resistance, to make its value proportional to area. The area conductance will be formulated below for several cases.

The area density of power loss is the value of power loss per unit area:

$$P_a = J^2/G_a \qquad \text{watts/m}^2 \qquad (11)$$

so the power loss for an area (A) is

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$$P = P_a A = AJ^2/G_a \qquad \text{watts} \qquad (12)$$

Various formulas are given in terms of skin depth (δ); they are useful if the ground properties happen to be expressed in such terms. The skin depth is (see p. 104 and Refs. 23, 43):

$$\delta = \sqrt{\frac{2}{\omega\mu_o\sigma}} = \sqrt{\frac{\lambda}{\pi\sigma R_c}} = \frac{1}{2\pi}\sqrt{\frac{\lambda}{30\sigma}} \qquad \text{meters} \qquad (13)$$

The simplest case of ground dielectric is that of large dissipation factor ($p >> 1$) so the conductive current is much greater than the capacitive current. The area conductance is

$$G_a = \sigma/c = \frac{\lambda}{\pi R_c \delta^2 c} = \frac{\lambda}{120\ \pi^2\delta^2 c} \qquad \text{mhos/m}^2 \qquad (14)$$

Another simple case is that of small dissipation factor ($p << 1$) so the conductive current is much smaller than the capacitive current (seldom encountered in ground). The capacitance and susceptance of unit area are

$$C = k\epsilon_o/c \qquad \text{farads/m}^2 \qquad (15)$$

$$B = \frac{k\epsilon_o\omega}{c} = \frac{2\pi k}{c\lambda R_c} = \frac{k}{60\ c\lambda} \qquad \text{mhos/m}^2 \qquad (16)$$

The area conductance, as defined above, is

$$G_a = B/p = \frac{2\pi k}{pc\lambda R_c} = \frac{k}{60\ pc\lambda} = \frac{(k\epsilon_o\omega)^2}{c\sigma} = \frac{4\pi^3 k^2\delta^2}{c\lambda^3 R_c} = \frac{\pi^2 k^2 \delta^2}{30\ c\lambda^3} \qquad \text{mhos/m}^2 \qquad (17)$$

In the general case,

$$G_a = B\,\frac{1+p^2}{p} \qquad \text{mhos/m}^2 \qquad (18)$$

If the conductance of the ground can vary over a wide range, the loss is greatest when $p = 1$ and G_a is least.

$$\min G_a = 2B \qquad (19)$$

ew

The area density of power loss has a corresponding value,

$$\max P_a = \frac{J^2}{\min G_a} = \frac{\pi c E^2}{k\lambda R_c} = \frac{E^2 c}{120\, k\lambda} \quad \text{watts/m}^2 \qquad (20)$$

In general,

$$P_a = \frac{2p}{1 + p^2} (\max P_a) \qquad (21)$$

As an example, take an area of <u>frozen ground</u> and determine the <u>greatest loss</u>:

$k = 4$
$p = 1$
$\lambda = 20$ Km
$A = 4$ Km^2
$h = 150$ m
$c = 3$ m
$P_o = 1$ Mw
p. 305(4) $V = 150$ Kv (sheet on top)
$E = 500$ v/m (= V/h x 1/2)

It is noted that the theoretical value of E is reduced to 1/2 to approximate the effect of grounded towers, which is partly the shielding effect of the towers toward the ground, and partly the extra height required to obtain the specified effective height.

(1)	$\sigma = pk/60\,\lambda = 3.3$ μ-mhos/m
(16)(19)	min $G_a = 2.2$ m-mhos/m
(10)	$J = 0.42$ ma/m^2
(20)	max $P_a = 80$ mw/m^2
(12)	max $P = 320$ Kw
	max $P/P_o = 0.32$

There follows the approximate computation based on the simple case of a sheet on top and a <u>uniform field</u> underneath:

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p.306(1) $p_o = 2.0$ mils
(7) max $p_a = 2.5$ mils
(8) max $p_a/p_o = 1.25$
max $P = P_o$ (max p_a/p_o) = 1250 Kw

Comparing the two values of power loss (max P), it appears that a reduction factor of about 1/4 is needed to reduce the latter value to the preceding value computed from the E-field on the ground. This factor is what would be expected from the assumed effects of the towers. Other effects are omitted in these examples.

The same area may be computed for normal (unfrozen) ground, with dissipation factor modified accordingly.

$\sigma = 0.7$ m-mho/m
(1) $p = 210 \gg 1$
(14)(11) $P_a = 0.76$ mw/m^2
(12) $P = 3.0$ Kw

Since p is large, the dissipation is about 2/p times the maximum value, or about 1/100 in this case. (This low a loss is unlikely to be realized because the ground near a buried conductor is found to have a much lower value of conductivity.)

On the surface of the ground, there are two alternative possibilities of a dissipative layer of dielectric. One is grass in the summer and the other is snow in the winter. Aside from the depth, the worst condition of either is that of low dielectric constant and unity dissipation factor. Taking a depth of one foot (c = 0.3 m) and the lowest limit of dielectric constant (k = 1), the maximum loss will be just 0.4 as great as that in the case of frozen ground computed above. A lesser value would occur with the somewhat higher dielectric constant met in practice.

NB 83, p. 51, 53-4, 60-2, 75-6, 81-2.

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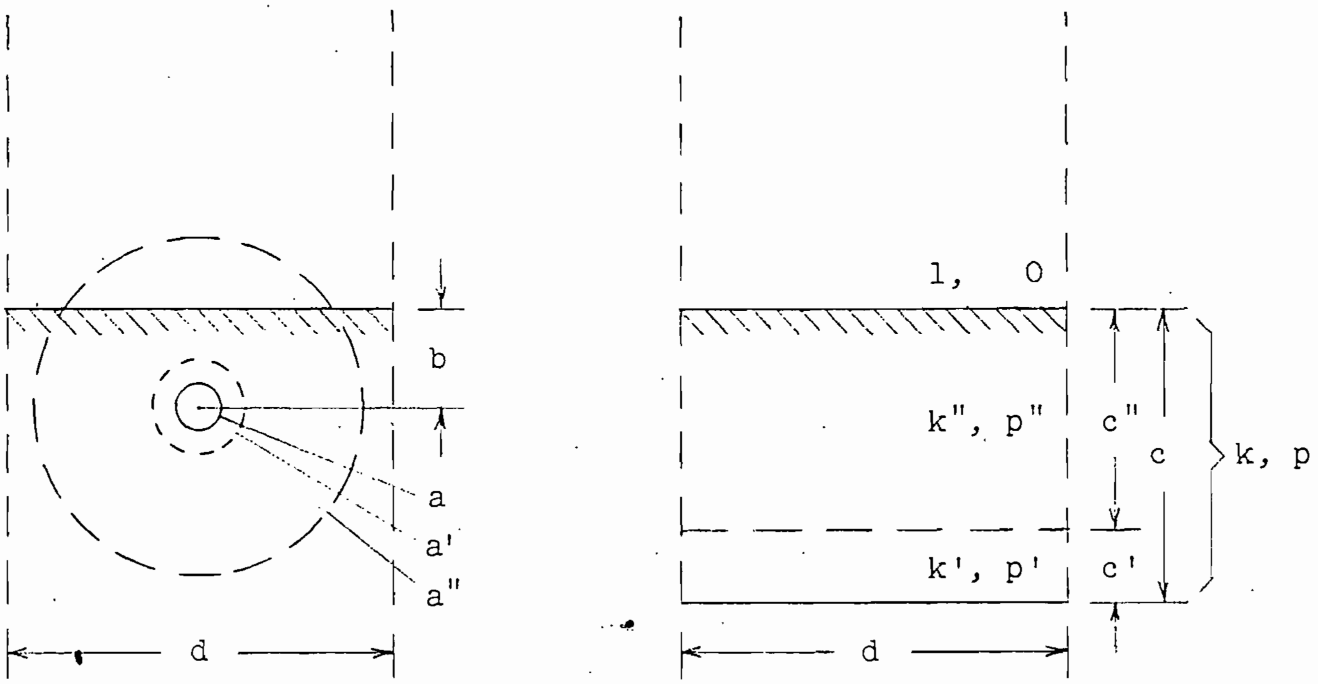
Buried Wires and Equivalent Plane.



(a) Buried wire (b) Equivalent plane

The vertical electric field is associated with a dielectric flux and capacitive current which are terminated at a horizontal grid of parallel wires buried in the ground. In some respects, this grid is equivalent to a horizontal plane conductor buried at a certain depth greater than the depth of the wires. The depth of the equivalent plane will be formulated, also the optimum depth of the wires which minimizes the depth of the equivalent plane.

The equivalence is based on several assumptions, as follows.

The wire and inner dielectric sheath are much smaller than the other dimensions:

$a < a' \ll a''$; $a < a' \ll b$; $a < a' \ll d$; in which

a = outer radius of buried wire

a' = outer radius of inner dielectric sheath around wire (if any)

a" = outer radius of dielectric boundary equivalent to ground surface

b = depth of buried wire (to center of wire)

d = width of space per wire in grid

= pitch of wires (between centers)

ew

The space under the surface is homogeneous, lossless dielectric; the dielectric constant in the inner sheath may differ from that in the remaining space.

The properties in the various regions are indicated in the diagram, and are symbolized as follows:

k = dielectric constant

$p = \sigma/k\epsilon_o\omega$ = dissipation factor, see p. 412(1)

In general, k is the apparent dielectric constant of the entire space under the surface; k' and k'' are the values for the inner and outer parts of this space, if different.

While the equivalence is based on pure dielectric, it gives a useful approximation for practical cases ranging between the limits of pure dielectric and pure conductor. These applications will be mentioned further on.

The equivalence of the wire grid and the plane conductor is based on equal amounts of electric energy in the respective spaces filled with like dielectric material. The formulas are derived by first imaging the wires above the surface, then mapping the field by conformal transformations that give uniform field below a distorted surface boundary, then integrating the cross-sectional area of this field as a measure of its energy, and finally locating the equivalent plane to give the same cross-sectional area below the actual surface.

If the ground has only one value of dielectric constant, the depth of the equivalent plane is

$$c = b + \frac{d}{2\pi}\left[\ln\frac{d}{2\pi a} + \frac{k-1}{k+1}\ln\frac{1}{1-\exp{-4\pi b/d}}\right] \qquad (1)$$

The 3 terms represent the respective effects of the depth of the wire, the field near the wire, and the bending of the field at the surface. All 3 terms are positive in the present conditions. The second term is usually the principal one. The third term decreases exponentially with increasing depth, since it is caused by the higher-mode interaction between the wire and its image.

ew

For any dielectric constant, there is a <u>depth</u> which is <u>optimum</u> in the sense of minimizing the electric energy and the corresponding depth of the equivalent plane. This optimum depth is

$$b = \frac{d}{4\pi} \ln \frac{3k-1}{k+1} \quad ; \quad \exp -4\pi b/d = \frac{k+1}{3k-1} \tag{2}$$

This depth ranges between the limits of zero for $k = 1$ (where the conditions are not met) to a maximum value for large dielectric constant. The corresponding depth of the equivalent plane is

$$c = \frac{d}{2\pi}\left[\ln \frac{d}{2\pi a} + \frac{1}{2}\ln\frac{3k-1}{k+1} + \frac{k-1}{k+1}\ln\frac{3k-1}{2k-2}\right] \tag{3}$$

This value is particularly useful because it is the minimum and also is a close approximation over a substantial range of actual depths which is likely to include the practical depth of the buried wire. The principal term is the first term, which is independent of the dielectric constant.

The limiting case of <u>high dielectric</u> ($k \doteq \infty$) gives the following simplified formulas.

$$c = b + \frac{d}{2\pi}\left[\ln\frac{d}{2\pi a} + \frac{1}{1 - \exp -4\pi b/d}\right] \tag{4}$$

The optimum depth of the wire, and the corresponding depth of the equivalent plane, are

$$b = \frac{d}{4\pi}\ln 3 = .0875\ d = d/11.43 \tag{5}$$

$$c = \frac{d}{2\pi}\left[\ln\frac{d}{2\pi a} + \frac{1}{2}\ln 3 + \ln\frac{3}{2}\right] = \frac{d}{2\pi}\left[\ln\frac{d}{2\pi a} + \ln\sqrt{\frac{27}{4}}\right]$$

$$= \frac{d}{2\pi}\ln\frac{d}{2\pi a}\sqrt{\frac{27}{4}} = \frac{d}{2\pi}\ln\frac{2.60\ d}{2\pi a} \tag{6}$$

This last form is very simple and is close enough for some practical cases.

ew

In practice, the dielectric has a dissipation factor whose value places it in one of the three cases to be described here.

If the dissipation factor is very small ($p \ll 1$) the field pattern is substantially that of a pure dielectric so the above formulas are valid.

If the dissipation factor is very large ($p \gg 1$) the field pattern is substantially that of a high dielectric ($k \doteq \infty$), so the above formulas (4) to (6) are applicable. The ground behaves as a conductor.

If the dissipation factor has any value, the field pattern is closely approximated by assuming a pure dielectric with the following value of k :

$$k\sqrt{1 + p^2} \tag{7}$$

Therefore this value may be used for k in formulas (1) to (3).

A circular boundary (of radius a'') around the wire may be defined, which is equivalent to the surface boundary, on the same basis as the plane is equivalent to the wire. In general, the radius of the equivalent circle is

$$a'' = a \exp 2\pi c/d \tag{8}$$

which may be computed from any of the formulas for c . For the limiting case of high dielectric ($k \doteq \infty$) and the optimum depth, the circumference of the equivalent circle is apparent in formula (6):

$$2\pi a'' = 2.60\ d \quad ; \quad 2a'' = 0.83\ d \quad ; \quad a'' = 0.41\ d \tag{9}$$

For any depth,

$$2\pi a'' = d\,\frac{\exp 2\pi b/d}{1 - \exp -4\pi b/d} = d\,\frac{\exp 4\pi b/d}{2 \sinh 2\pi b/d} > 2.60\ d \tag{10}$$

It appears that the circumference of the equivalent circular boundary is several times the wire spacing.

ew

If there is a sheath of different dielectric (of outer radius a') closely surrounding the wire, there are two dielectric constants, one (k') in this sheath and another (k") outside. The depth including the equivalent plane boundaries is separated into two corresponding parts:

$$c' = \frac{d}{2\pi} \ln \frac{a'}{a} \;;\quad c'' = c - c' = \frac{d}{2\pi} \ln \frac{a''}{a'} \tag{11}$$

whose total is

$$c = c' + c'' = \frac{d}{2\pi} \ln \frac{a''}{a} \tag{12}$$

Each of the two parts may be treated separately in terms of the planes.

Since c is usually of the same order as d , the following example is developed as one in which these two dimensions are equal. The wire is buried at the optimum depth.

Let	$kp \gg 1$
(8)	$a''/a = \exp 2\pi = 540$
(9)	$a''/d = 0.41 \;;\; a/d = 0.41/540 = 1/1300$
Let	$c = d = 3$ m
Then	$a'' = 1.23$ m
	$a = 3/1300$ m $= 2.3$ mm

Departure from optimum depth, in the ratio of 1/2 or 2, increases a" in the ratio of about 1.25, and increases c by about .035 d. In this example, the relative change of c is only 1/28. This verifies that the formulas based on optimum depth are a close approximation over a substantial range of depth.

The values of c (or c' and c") are intended to be utilized for computing losses in the preceding section (p. 411-417).

NB 83, p. 51, 53-6, 60-2, 64-74, 83-5.

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Length of Buried Wire for E-Field.

The total length of buried wire and the total current in the antenna determine the total power of the E-field losses in a simple relationship that is useful for rough computations. It is based on the assumption of uniform field (E) over an area sufficient to carry the entire current. Then a uniform pitch of parallel wires is best. Some reasonable value of the pitch (d) is assumed, and the formula is noncritical to this value.

The total capacitance and susceptance of the total length of buried wire are expressed in terms of the diagrams on p. 411 and 418.

$$C = k\epsilon_o ld/c \tag{1}$$

$$B = \frac{2\pi ldk}{\lambda c R_c} \tag{2}$$

in which

C = effective total capacitance between surface and total length of buried wire (farads)
B = susceptance of C (mhos)
c = depth of equivalent plane (meters)
d = pitch of parallel wires (meters)
l = total length of wire (meters)
k = dielectric constant in ground

It is noted that c and d are of the same order in practice. The total current is divided between this susceptance and the parallel conductance, which are related by the dissipation factor (p) of the dielectric.

Going to the simple case of high dielectric ($k >> 1$) and optimum depth ($b = d/11.43$, p. 420), the ratio c/d is determined, p. 420(6). Further assuming the worst conductance ($p = 1$, p. 413) it appears that the following value is the effective series resistance which carries the total current in the antenna condenser:

$$R = 1/2B = R_c \frac{\lambda}{8\pi^2 lk} \ln\frac{2.6\ d}{2\pi a} = \frac{15\ \lambda}{\pi lk} \ln\frac{2.6\ d}{2\pi a} \tag{3}$$

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If this resistance carries the current required to radiate a certain amount of power (P_o), given on p. 304(3), the relative amount of dissipated power (P) is expressed by the ratio,

$$P/P_o = \frac{3(\lambda/2\pi)^3}{4h^2 lk} \ln \frac{2.6\ d}{2\pi a} \tag{4}$$

Conversely, if the dissipation is to be limited to this ratio, the required total length of wire is

$$l = \frac{P_o}{P} \frac{3(\lambda/2\pi)^3}{4h^2 k} \ln \frac{2.6\ d}{2\pi a} \tag{5}$$

To express the power dissipation in either part of the ground space (p. 418), substitute for the logarithm in (5):

$$\ln a'/a \quad \text{or} \quad \ln a''/a' \tag{6}$$

Part of the current of the antenna condenser is carried to the ground wires through the grounded towers and guy wires, so only the remaining fraction of the current reaches the ground surface to cause E-field losses. The dissipated power (4) is proportional to the square of this fraction. Or, for a specified amount of dissipated power, the required length of wire (5) is proportional to the square of this fraction.

Taking an example of frozen ground, similar to that on p. 416, 422, and computing the required length of wire:

λ = 20 Km	k = 4
h = 150 m	p = 1
d = 3 m	$\ln 2.6\ d/2\pi a = 2\pi$
a = 2.3 mm	$P/P_o = 0.32$
	(5) l = 5300 Km

If 1/2 the current is carried to the ground wires by the towers and 1/2 flows into the ground surface, the required length is 1/4 as great, or 1300 Km.

NB 83, p. 25-7, 62.

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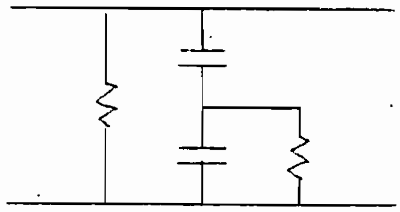
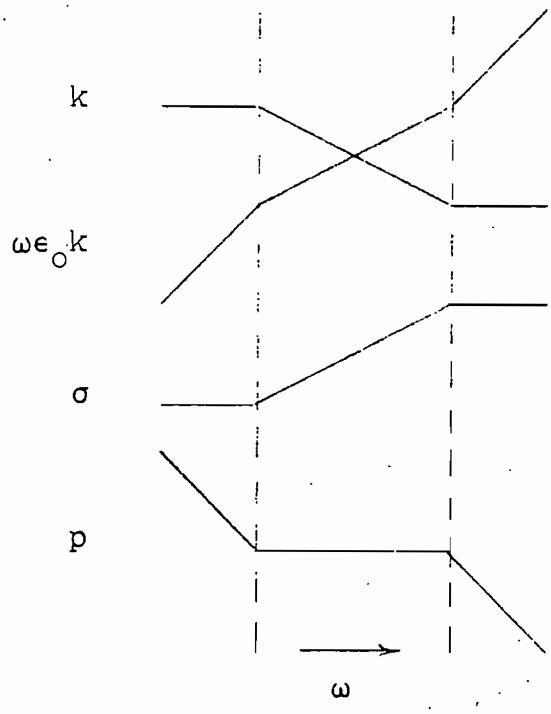
Dielectric Properties of Snow.



(a) Equivalent network



(b) Variation of properties

Recent tests at NEL have yielded valuable information on the dielectric properties of snow over a wide frequency range (1 - 200 Kc) including the VLF operating range (14 - 30 Kc).

As might be expected, the dielectric has parallel conducting paths, some of which are continuous and others discontinuous. Diagram (a) shows the simplest network that simulates this situation, and (b) shows diagramatically how the various properties vary with frequency (on logarithmic scales).

k = dielectric constant

σ = conductance of unit cube

$p = \sigma/\omega\epsilon_o k$ = dissipation factor

$\omega\epsilon_o k$ = susceptance of unit cube

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In the transition region of the frequency scale, the properties shift from one to the other of two limiting levels or laws. In most cases, the dissipation factor has a positive (reverse) slope in the middle of the transition region, indicating that the slope of the susceptance is exceeded by that of the conductance. The transition region is centered in the vicinity of 10 - 15 Kc, near the lower limit of the VLF operating range, where the dielectric losses are most harmful.

The two samples that were tested are described as "fresh snow" and "packed snow". The average of tests at -1 to -18°C (or 30 to 0°F) is approximately the following:

Material	max k	min k	k (15 Kc)	p
Fresh snow	7	1.6	3.4	0.6
Packed snow	25	2.5	9	0.9

The values and the ratio of k are both greater for packed snow, indicating greater density and greater conductivity. At 15 Kc, which is near the middle of the transition region, there is an intermediate value of k and a dissipation factor somewhat less than unity. The latter is an inherent property of this type of network (a). For the same depth, and more so for the same mass, the fresh snow causes greater E-field loss because it is less dense and hence a lower dielectric.

The minimum value of dielectric constant (at high frequencies) is somewhat less than that of ice (k = 3.15) as would be expected. See references (63), (66), (71).

Referring to the previous example (p. 416-7) we may reconsider the E-field loss of one foot of fresh snow. The loss is equal to that of a certain "worst" dielectric (k = 4, p = 1); it is 0.1 as great as that computed for frozen ground (0.1 x 1/4 x 1250 = 30 Kw). The same depth of packed snow causes even less loss (4/9 x 30 = 13 Kw). Therefore one foot of snow causes an additional loss very much less than that caused by frozen ground.

NB 83, p. 28, 86-8.

ew

Dielectric Properties of Frozen Soil.

Recent tests by Smith Electronics have yielded valuable information on the dielectric properties of frozen soil at 15 Kc. The tests were made between coaxial conductors, the inner one being No. 6 wire and the outer one much larger. The two samples tested were from Maine, and are described as "No. 3, black peat soil" and "No. 4, gray sandy loam".

The behavior is qualitatively similar to that of snow, simulated by the equivalent network on p. 425.

The two samples yielded nearly the same dissipation factor in the frozen condition, increasing at lower temperatures as follows.

Temperature		Dis. Factor (p)	Q = 1/p	Diel. Cst. (k)
$-1^{\circ}C$	$30^{\circ}F$	10	0.10	40
-7	20	4.4	0.23	10
-13	10	2.9	0.35	6
-18	0	2.2	0.45	4
-23	-10	1.9	0.52	3

The energy storage factor ($Q = 1/p$) increases at a nearly uniform rate with decreasing temperature. This means that the effective conductivity is decreasing this much more than the effective dielectric constant.

In the transition through the freezing point ($0^{\circ}C$ or $32^{\circ}F$), the behavior is anomolous, indicating that most of the contained moisture is a very dilute solution.

In the above table, each value of dielectric constant is the ratio mean of two values differing in a ratio as great as 2 or 3; the mean is regarded as representative.

Recently there was a choice of values to be used as a basis for design of the ground system. The chosen values were based on these tests, at about $10^{\circ}F$, as follows: $k = 6$; $p = 2.75 = 1/\tan 20^{\circ}$. The loss in this dielectric is the same as if $k = 9.4$ and $p = 1$, so the power loss is about 4/9.4 or 0.43 of that computed in the example on p. 416-7 (about 130 Kw).

ew NB 83, p. 39.

Symbols.

Supplement to p. 103 and 410; with one change from I_1 to I_c .

$2\pi r/\lambda$ = distance angle (radians)
$2\pi h_o/\lambda$ = effective-height angle (radians)
C_o = capacitance of antenna condenser (farads/meter^2)
I_c = total radial current of radiation term (constant, amperes)
I_c = 158 amp if P = 1 Mw
J = area density of vertical current (amperes/meter^2)
G_a = series component of area conductance between surface and buried conductor (mhos/meter^2)
P_a = J^2/G_a = area density of power dissipated (watts/meter^2) elec
a' = outer radius of inner dielectric sheath around wire (if any) (meters)
a" = outer radius of dielectric boundary equivalent to ground surface (meters)
1/d = density of buried wire (number per meter)
δ = skin depth in the ground (meters)
k = dielectric constant of the ground
p = $\sigma/k\epsilon_o\omega$ = dissipation factor in the ground elec
p_a = dissipation factor of antenna condenser

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INCOMPLETE

Report 911

ULF ANTENNAS AND PROPAGATION

By Harold A. Wheeler

To RCA Laboratories
Purchase Order N-82(15103-0)-14
1959 SEP 2 Job 399

~~Note: Some parts of this report will be classified CONFIDENTIAL~~.

cyl. field, interception area -
ionosphere "gain", cyl/sphere power. (NB 101 p.12) p.10 X

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Contents

This is a cumulative report for collecting information developed in the course of studies and consultation on the PANGLOSS project for electromagnetic communication with submerged submarines, especially by means of ULF waves. The following list will be brought up to date and reissued with each release of additional pages.

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Radiation between parallel planes.

Toward ULF fields and waves, the ground and ionosphere can be approximated by perfectly conducting concentric spheres. These in turn can be approximated by parallel planes out to distances much less than a quadrant of the earth's circumference (quadrant is about 10,000 Km or 10 Mm).

Single-mode propagation is assumed, which is effective below about 2 Kc. Attenuation in propagation is ignored, which is justified for approximations under the stated conditions.

Fig. 2 shows sending antennas of C and L types, radiating between parallel planes.

The radiation power factor of any reasonable size of antenna is so small that it has no practical effect as compared with the heat power factors of the antenna reactance. Therefore the more useful formulas are those for the radiation field at a distance, in terms of the current in the sending antenna.

Symbols (rationalized MKS units).

f	=	frequency
λ	=	wavelength
$\lambda/2\pi$	=	radianlength
r	=	distance from sender to receiver
h	=	effective height
g	=	effective horizontal length of antenna (L type)
A	=	effective area of antenna ($A_m = gh_m$)
I	=	current in sending antenna (RMS)
R	=	radiation resistance
R_c	=	120π = 377 ohms = wave resistance of free space
E	=	$H R_c$ = electric field intensity at receiver (RMS)
H	=	magnetic field intensity at receiver (RMS)
sub-i	=	ionosphere
sub-e	=	sending antenna of C type
sub-m	=	sending antenna of L type

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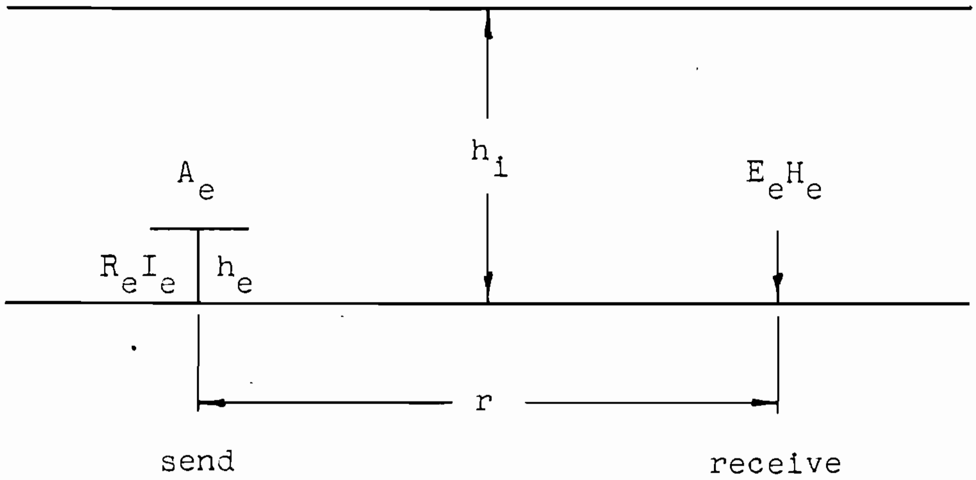
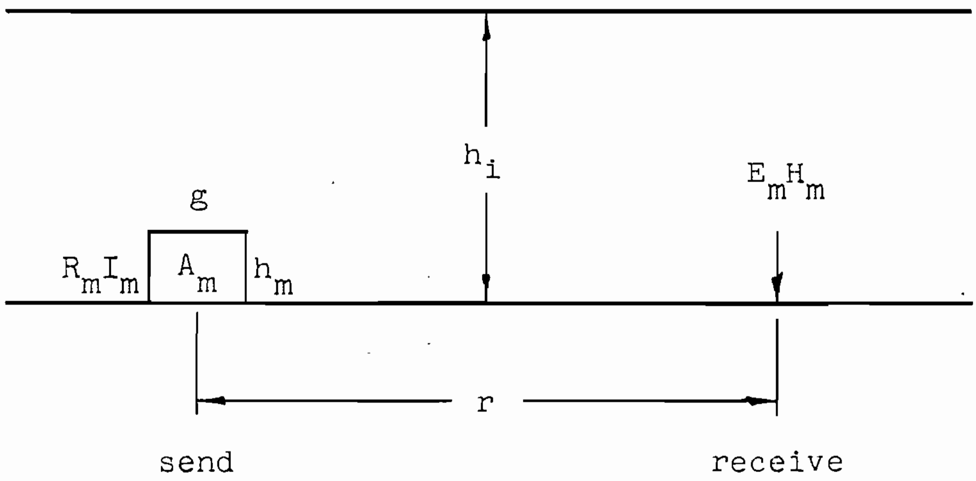


(a) C type.



(b) L type.

Fig. 2 - Radiation from antennas of C and L types, between parallel planes.

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Conditions.

$h_e \ll h_i$; $h_m \ll h_i$; $g \ll \lambda/2\pi$
$h_i < \lambda/2$; $\lambda/2\pi < r$

Formulas for field intensity.

From C type: $$H_e = \frac{I_e}{2\sqrt{r\lambda}} \frac{h_e}{h_i} \qquad (1)$$

From L type: $$H_m = \frac{I_m}{2\sqrt{r\lambda}} \frac{2\pi g h_m}{\lambda h_i} \qquad (2)$$

For the cylindrical wave between parallel planes, it is noted that H varies with $1/\sqrt{r\lambda}$, unlike $1/r$ for the spherical wave in free space.

It is preferable to use H (amperes/meter) rather than E (volts/meter) because H is continuous at the air-to-sea boundary and is directly utilized in the small loop antenna of the usual submarine receiver.

Formulas for radiation resistance.

C type: $$R_e = \frac{1}{4} R_c \frac{2\pi h_e^2}{\lambda h_i} = 60\pi^2 \frac{h^2}{\lambda h_i} \qquad (3)$$

L type: $$R_m = \frac{1}{8} R_c \frac{(g h_m)^2}{h_i(\lambda/2\pi)^3} \qquad (4)$$

Between parallel planes, R_e varies with $1/\lambda h_i$, unlike $1/\lambda^2$ for free space. The radiation resistance in both cases is increased by the presence of the ionosphere.

The example on pp. 3-7 above is an application of these formulas, among other considerations.

(4) H. A. Wheeler Notebook 95, pp. 55, 65.

(5) H. A. Wheeler, "A vertical antenna made of transposed sections of coaxial cable", IRE Convention Record, vol. 4, pp. 160-4; 1956. (Radiation resistance of a vertical wire between horizontal planes.)

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Earth-shell resonance.

The hollow cavity between ground and ionosphere has certain frequencies of resonance, assuming perfect conducting boundaries with free space between. The n-th mode of resonance corresponds with a free-space wavelength as follows:

$$\lambda_n = \frac{2\pi a}{\sqrt{n(n+1)}} \tag{5}$$

in which $2\pi a$ is the mean circumference in the shell. The ionosphere height is so small that, for practical purposes, $2\pi a = 40{,}000$ Km = 40 Mm , the earth's circumference at the surface. The resonance frequencies for the first few modes are computed as follows:

$$f = 10.6\ ,\ 18.4\ ,\ 26.0\ ,\ 33.5\ ,\ \text{etc.} \tag{6}$$

For the higher modes, the spacing approaches 7.5 cycles/second.

Some observations have indicated that the actual resonance occurs at a substantially lower frequency. Recent studies by the writer have indicated that this should be so, in view of the graded conductivity in the space where the air is intended to act as a dielectric. This conductivity is caused mainly by ionization from cosmic radiation. The effective height for electric field becomes substantially less than that for magnetic field, the latter height being closer to the ionosphere as observed at higher frequencies (VLF). The result is a reduction of the velocity and therefore of the resonance frequencies, by a fraction which is smaller at lower frequencies. Also, the electric-field losses become the principal cause of attenuation in ULF wave propagation around the earth.

(6) H. A. Wheeler Notebook 95, pp. 34-60.

(7) Schumann, papers #4, 5 and 6 listed in letter from C. Polk, Aug. 18, 1959.

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Air conductivity variation with height.

Air conductivity increases steadily from a very low value at low altitude to a maximum value at a high altitude in excess of 100 Km. It depends on ionization caused by some form of radiation. At low altitude, the main cause is cosmic rays, which are nearly constant over the daily cycle. At high altitude, the daily cycle of solar rays causes a large difference between night and day.

Ignoring the earth's magnetic field, which may or may not be permissible, the principal effect of ionization is the resulting conductivity. At a high altitude, it provides a conductive boundary for the alternating magnetic field. At a lower altitude, it may provide an upper boundary for the electric field; this occurs at ULF. The latter effect lowers the wave velocity and is the principal cause of attenuation in propagation.

Fig. 3 shows some values and trends in the variation of air conductivity. At low altitude, the minimum value is about 2×10^{-14} mho/meter. The increase up to 20 Km has been measured by Gish, as plotted. The only definite value shown for high altitude is that of Wait, based on VLF daytime tests and the assumption of a step of conductivity (as diagramed). The "day" line is based on Gish and Wait. The "night" line is based on conjecture. There is no attempt to include the wavy variations that provide reflecting levels at high frequencies; they are ignored for ULF purposes.

There are noted the probable levels of the effective boundaries at 10 cycles/second. The E boundary is shown about half as high as the H boundary, which would reduce the wave velocity to about 0.7 of the free-space velocity.

Fig. 4 shows the transmission-line analog of the space occupied by ULF propagation. The series impedance represents the magnetic field and the skin effect at the upper boundary. The shunt admittance represents the electric field and the shunt conductance which destroys it at high altitude. Since the ionosphere height is much less than a half-wavelength (at frequencies much less than 2 Kc) the separate formulation of electric and magnetic fields is justified.

The straight lines represent a conductivity variation which is exponential with height. This is simply described and its effect

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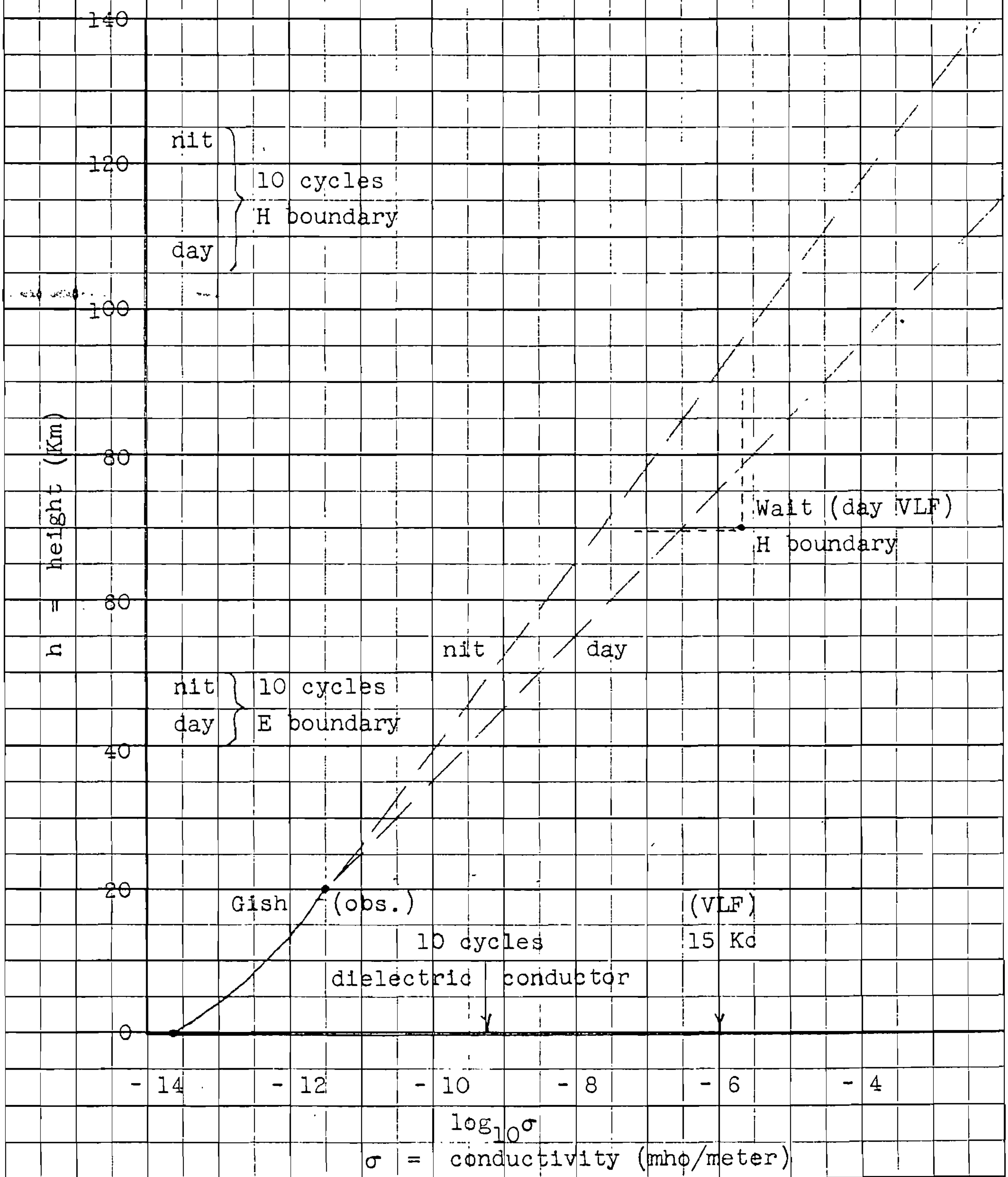

Fig. 3 - Air conductivity variation with height.

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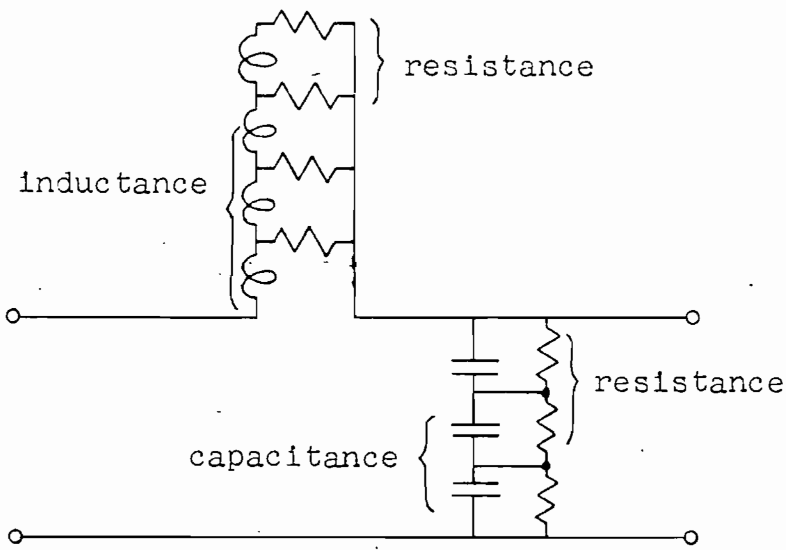

Fig. 4 - Transmission-line analog of space between ground and ionosphere.

is easily computed. An accurate description is essential only in the boundary region, where further measurements are needed. In the meantime, Fig. 3 gives a reasonable basis for estimates.

(8) H. A. Wheeler Notebook 95, p. 12-68.

(9) J. A. Chalmers, "Atmospheric Electricity", Pergamon Press, 1957. (Conductivity of the air, Gish measurements up to 20 Km, p. 135-138. Text has numerous errors and inconsistencies in units and values.)

(10) J. R. Wait, "The mode theory of VLF ionospheric propagation for finite ground conductivity", Proc. IRE, vol. 45, p. 760-7; June, 1957. (Observations at 16-18 Kc are consistent with effective height of 70 Km, ionosphere conductivity step of 2 μ-mho/meter.)

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Limitations in sending antenna of C type.

Symbols (rationalized MKS units, see p. 8).

ω	=	$2\pi f$ = radian frequency
ϵ_o	=	electrivity of free space
$\omega\epsilon_o$	=	$2\pi/\lambda R_c$ = susceptivity of free space
R_c	=	377 ohms = wave resistance of free space
R_e	=	radiation resistance, p. 10(3)
I_e	=	current (RMS)
P_e	=	radiation power
h_e	=	effective height
h_i	=	height of ionosphere
A_a	=	area of conductor surface (wires above ground)
E_a	=	average voltage gradient on conductor surface (RMS)
E_{ac}	=	2.1 Kv/mm (RMS) for breakdown between parallel planes in air, at sea level, at room temperature
sub-a	=	conductor (C type)

Current in terms of voltage gradient on wires.

$$I_e = \omega\epsilon_o A_a E_a = \frac{2\pi A_a E_a}{\lambda R_c} \tag{7}$$

Voltage gradient on wires.

$$E_a = \frac{\lambda R_c I_e}{2\pi A_a} \tag{8}$$

Conductor area.

$$A_a = \frac{\lambda R_c I_e}{2\pi E_a} \tag{9}$$

Radiation power.

$$P_e = R_e I_e^2 = \frac{E_a^2 h_e^2 A_a^2}{4R_c h_i (\lambda/2\pi)^3} \tag{10}$$

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Effective height x conductor area.

$$h_e A_a = \frac{2}{E_a} \sqrt{P_e R_c h_i (\lambda/2\pi)^3} \tag{11}$$

The C type of antenna is similar to that of Figs. 1(a) and 2(a).

These formulas give the limitations imposed by the voltage gradient on the conductor wires. (Reference 3, p. 7.) This gradient must not exceed some specified value at which the corona discharge would become excessive. This condition might be determined by the resulting radio noise in nearby populated areas, or by arcing to nearby metal structures. The former has been the controlling factor on power lines. These effects are more severe during rainfall, fog or icing.

The Maine VLF antenna is conservatively designed for a gradient having a maximum value of 0.83 Kv/mm and an average value of 0.50 Kv/mm. It is intended to avoid any appreciable corona, even during rainfall.

The permissible average gradient E_a is determined by several factors, the first of which is its value in air between parallel planes. It is decreased by each of the following factors:

Lesser air pressure (higher altitude).
Corrugated surface (stranded wire).*
Water drops or icing.*
Crowding of charge distribution.*

It is increased by each of the following factors:

Curvature of the wire.*
Lower frequency.
Tolerance of corona.
Covering of insulating material.

The indicated items (*) are present in the Maine antenna; it is designed for an average gradient less than 1/4 the nominal value.

The most difficult problem is the excess gradient caused by crowding of the charge in some places on the wires. This may be caused by the exposed location of the wire or by its proximity to a grounded structure. After some attention to this problem, the Maine antenna still had a maximum gradient 1.66 times the average value. This is probably the first limitation on the amount of power it can handle.

The reactive power is determined by the concentration of the wires, with proportionate decrease of capacitance and increase of voltage. This voltage must be handled by the supporting insulator strings.

The reactive power may be supplied by a tuning inductor. At ULF, this inductor may take most of the real power, which can be decreased by increasing the size of the inductor. Alternatively, a "rotary inductor" may be used, or the generator may be designed for a capacitive load.

The height and conductor area are the practical requirements of the structure, because they determine the wind and ice loading. Therefore their product (11) is the principal specification.

The effective height is decreased by downleads and by proximity to grounded towers and guy wires.

The radiation of this antenna is omnidirective in the horizontal plane. If used in a spherical-shell resonator, it is located at one of the two poles of maximum electric field.

A numerical example (based on the Maine antenna) is found on p. 4-5 above. Here is further information on this example:

Conductor area	A_a	= 9,000 meters^2
Average gradient	E_a	= 0.70 Kv/mm
Max. gradient		1.15

(11) H.A.W. Notebook 95, p. 98-116.

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Limitations in sending antenna of L type.

Symbols (rationalized MKS units, see p. 8).

R_c	=	377 ohms = wave resistance of free space
R_m	=	radiation resistance, p. 10(4)
I_m	=	current (RMS)
P_m	=	radiation power
h_m	=	height of loop
g	=	length of loop
gh_m	=	area of loop
h_i	=	height of ionosphere
P_b	=	power to conductor heating
R_b	=	resistance of conductor
A_b	=	area of conductor surface (wires above and below)
b	=	effective thickness of conductor
σ	=	conductivity of conductor
σ_c	=	58 megamhos/meter = conductivity of copper
sub-b	=	conductor (L type)

Conductor resistance.

$$R_b = P_b/I_m^2 = \frac{(2g + 2h_m)^2}{\sigma b A_b} \qquad (12)$$

Conductor power.

$$P_b = R_b I_m^2 = \frac{I_m^2\,(2g + 2h_m)^2}{\sigma b A_b} \qquad (13)$$

Conductor power density.

$$P_b/A_b = \frac{I_m^2\,(2g + 2h_m)^2}{\sigma b A_b^2} \qquad (14)$$

Conductor volume.

$$bA_b = \frac{(2g + 2h_m)^2}{\sigma\, R_b} = \frac{I_m^2\,(2g + 2h_m)^2}{\sigma\, P_b} \qquad (15)$$

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Current.

$$I_m = \frac{1}{2g + 2h_m} \sqrt{P_b \sigma b A_b} \tag{16}$$

Radiation power.

$$P_m = R_m I_m^2 = \frac{1}{8} P_b R_c \sigma b \frac{(gh_m)^2 A_b}{(2g + 2h_m)^2 h_i (\lambda/2\pi)^3} \tag{17}$$

Radiation efficiency.

$$P_m/P_b = R_m/R_b = \frac{1}{8} R_c \sigma b \frac{(gh_m)^2 A_b}{(2g + 2h_m)^2 h_i (\lambda/2\pi)^3} \ll 1 \tag{18}$$

Height x conductor area.

$$h_m A_b = \frac{2g + 2h_m}{g} \sqrt{\frac{8 P_m h_i (\lambda/2\pi)^3}{(P_b/A_b) R_c \sigma b}} \tag{19}$$

This formula (19) is somewhat comparable with (11) for the C type.

The L type of antenna is similar to that of Figs. 1(b) and 2(b), comprising a single-turn loop of several wires in parallel. The upper half of the conductor area A_b is roughly comparable with the entire area A_a in the C type. In commenting on the power density on the loop wires, it is assumed that the upper wires (in the air) are the limiting factor.

These formulas give the limitations imposed by the conductor power density on the surface of the wires. This determines the amount of heat to be dissipated, and the resulting temperature rise. While the ultimate capability is limited by the permissible maximum temperature of the wires, the available power may be insufficient to utilize this capability.

As an example of the power density that may be permissible, take a horizontal wire that would assume a temperature of 50° C under hottest ambient conditions. A rise of about 50° (to 100° C) might be caused by

$$P_b/A_b = 2 \text{ Kw/m}^2$$

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(The value to be used for de-icing the Maine antenna is 2.5 Kw/m^2, which would be applied only in freezing weather.) Since the major part of the heat dissipation from the wire is by convection, even in still air, the condition of the surface is not important.

If the temperature rise is the ultimate limitations, it should be equalized on all wires, which is easily approximated. Any insulating covering should be thin enough to offer little thermal resistance.

The reactive power is determined by the concentration of the wires, with proportionate increase of inductance and voltage. This voltage must be handled by the supporting insulators; at UHF it is much less than the voltage required by the C type.

The reactive power may be supplied by a tuning capacitor. Alternatively, a "rotary capacitor" may be used, or the generator may be designed for an inductive load.

As in the C type, the height and conductor area are the practical requirements of the structure, because they determine the wind and ice loading. Therefore their product (19) is the principal specification.

Unlike the C type, the effective height of the loop is substantially the average height of the horizontal top wires, because it is not appreciably reduced by downleads and by proximity to grounded towers and guy wires.

The radiation of this antenna has a figure-8 pattern in the horizontal plane, so only half as much radiated power is required for the same range between parallel planes. If used in a spherical-shell resonator, it is located on the equator of maximum magnetic field with the loop oriented in the plane of the poles.

A numerical example is found on p. 5-6 above. Here is further information on this example:

Conductor area	$A_b = 2A_a = 18{,}000$ meter^2
Power density	$P_b/A_b = 75$ w/m^2
Conductivity (15%)	$\sigma = 8.7$ megamho/m
Effective thickness	$b = 2.9$ mm

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(The last two values are queried, but are roughly correct.) In accordance with comments above, the power could be increased about 27 times within the limit of over-heating.

(11) H.A.W. Notebook 95, p. 98-116.

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Comparison of antennas of C and L types.

A formula will be given for comparison of the ultimate limitations of the C and L types of antennas, as given in the two preceding sections. The comparison is made in terms of the field-strength ratio at a receiver.

Sender antenna of C type:

H_e = magnetic field intensity at receiver (RMS)
h_e = effective height of antenna
A_a = conductor area of wires
E_a = voltage gradient on conductor (RMS)

Sender antenna of L type:

H_m = magnetic field intensity at receiver (RMS)
g = length of antenna loop
h_m = height of antenna loop
A_b = conductor area of wires
P_b = power input to conductor
b = effective thickness of conductor
σ = conductivity of conductor
R_c = 377 ohms

Field-strength ratio at receiver:

$$\frac{H_m}{H_e} = \frac{g}{2g + 2h_m} \frac{h_m A_b R_c}{h_e A_a E_a} \sqrt{\sigma b (P_b/A_b)} \tag{20}$$

This ratio is unity in the examples of C and L types given above. This formula shows how it depends on the factors which have been chosen to express the ultimate limitations of the two types. It is notable that this ratio is independent of frequency.

To remove some of the factors that contribute little to the difference of the two types, insert the following from the above examples:

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$h_m/g = .058$
$h_m/h_e = 1.4$
$A_b/A_a = 2$
$(E_{ac} = 2.1$ Kv/mm RMS for air breakdown)
$(\sigma_c = 58$ megamho/m for copper)
$(P_{bc}/A_b = 2$ Kw/m^2 for permissible heating)

$$\frac{H_m}{H_e} \doteq \frac{2.55}{(E_a/E_{ac})} \sqrt{(1000\ b)(\sigma/\sigma_c)(P_b/P_{bc})} \qquad (21)$$

This ratio is 5.1 if b = 4 mm and the three ratios are unity to saturate the corresponding limits, the latter being only approximately practicable.

(11) H.A.W. Notebook 95, p. 98-116.

Near fields between parallel planes.

The near field of an antenna is of interest if the distance is less than the radianlength. This is true of long-range communication at unusually low frequencies. Also it is true of short-range interference from lightning (C type of radiator). Formulas will be given first for the C type of antenna and then for the L type.

Referring to Fig. 2(a), there are three modes in the field from the C type. The field at the receiver will be formulated for each mode, also each distance of transition from one mode to the next.

Spherical near field: $h_e < r < h_1 < \lambda/\pi^2$

$$H_e = \frac{I_e h_e}{2\pi r^2} \tag{22}$$

Cylindrical near field: $h_1 < r < \lambda/\pi^2$

$$H_e = \frac{I_e h_e}{2\pi r h_1} \tag{23}$$

Cylindrical far field (radiation): $h_1 < \lambda/\pi^2 < r$ (?)

$$H_e = \frac{I_e h_e}{2\sqrt{\lambda r}\, h_1} \tag{1}$$

As usual, these are closer approximations under conditions further from the transition limits. Each of these limits is defined as the distance at which the formulas for two adjacent modes give equal values for the field.

Referring to Fig. 2(b), there are likewise three modes in the field from the L type, in the two directions in the plane of the loop. These will be formulated in the same manner. The coefficient 0.86, which appears in one of the transition limits, represents $(2/\pi)^{1/3}$.

Spherical near field: $g + h_m < r < h_1 < 0.86\ \lambda/2\pi$

$$H_m = \frac{I_m g h_m}{2\pi r^3} \tag{24}$$

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Cylindrical near field: $h_i < r < 0.86\ \lambda/2\pi$

$$H_m = \frac{I_m g h_m}{2\pi r^2 h_i} \tag{25}$$

Cylindrical far field (radiation): $h_i < 0.86\ \lambda/2\pi < r$?

$$H_m = \frac{\pi I_m g h_m}{\lambda\sqrt{\lambda r}\ h_i} \tag{2}$$

The L type has another set of only two modes, in the directions of the axis of the loop. The parallel planes prohibit any radiation mode in these directions.

Spherical near field: $g + h_m < r < 2h_i < \lambda$

$$H'_m = \frac{I_m g h_m}{\pi r^3} \tag{26}$$

Cylindrical near field: $2h_i < r$

$$H'_m = \frac{I_m g h_m}{2\pi r^2 h_i} \tag{27}$$

Attention is directed to the 1:2 ratio of the two formulas for the spherical near field, and their equality for the cylindrical near field.

In general, the corresponding transition limits differ slightly in the different sets of modes. In each set of three modes, it is possible for the intermediate mode (cylindrical near field) to collapse while still maintaining $h_i < \lambda/2$, the condition for single-mode radiation; then there is a new limit of transition between the two remaining modes.

(13) H.A.W. Notebook 95, p. 141, 150.

(14) H.A.W. Notebook 99, p. 3-4.

(15) C. A. Martin of RCA called the writer's attention to the significance of the cylindrical near field in the directions of the axis of the loop.

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(16) E. C. Jordan, "Electromagnetic Waves and Radiating Systems", Prentice-Hall; 1950. (Magnetic field of parallel wires, p. 88-89. Cylindrical electromagnetic field of a single long wire, p. 370-378.)

(17) H. A. Wheeler, "A vertical antenna made of transposed sections of coaxial cable", IRE Convention Record, vol. 4, part 1, p. 160-164; 1956. (Field of vertical wire between two horizontal planes.)

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Lightning.

Since lightning is the principal cause of ULF radio noise, its occurrence and its characteristics are relevant to communication predictions. The essential information is scattered and not well summarized for present purposes. The following abstract is selectively gleaned from a few of the recognized authorities. After introducing the terminology and concepts, some statistics will be given on the basis of some reports and rough estimates.

A thunderstorm occurs in a limited area where the cloud formations include an interplay of ice and water particles. This seems to be essential to the separation of charges in different parts of a cloud. Just before a discharge between cloud and earth, with which we are here concerned, the lower part of the cloud is usually negatively charged. During the discharge, this negative charge is neutralized by positive charge from the earth. (This positive charge is continuously being replenished by leakage currents between earth and ionosphere, through the small conductivity caused by ionization of the air by cosmic rays.)

A single cloud is discharged by a single flash, which is over in a second. This discharge is accomplished in one or more strokes from different parts of the cloud. For present purposes, each stroke has a sudden pulse of high current, followed by a slower discharge, the latter including most of the total charge. This is significant in determining the field components at lower frequencies.

In each stroke, the initial pulse of current at the ground is greater and shorter than the average over the length of the stroke. The latter is effective for developing the field at a distance greater than the length.

Thunderstorms (typical or average values):

Duration of storm	50 min = 3000 sec
Number of strokes per storm	200
Frequency of storms over the earth	44,000 per day = 0.5 per sec
Frequency of strokes over the earth	100 per sec
Frequency of strokes during a storm	4 per min = 1/15 per sec
Average number of strokes per flash	2
Frequency of flashes during a storm	2 per min = 1/30 per sec
Frequency of thunderstorms in any one location	
Middle U.S.	50 per year = 0.14 per day
U.S. extremes	1-100
Ocean ("infrequent")	2 per year = .0055 per day
Earth, average	10 per year = .027 per day
Average area of a storm, over the earth	$80\ Km^2 = \pi\ (5.0\ Km)^2$
Local time during which thunderstorms are common	Fourth quarter of the day (18-24 hrs)

Lightning strokes (typical or median values):

Peak current at ground	20 Ka (extreme, 200 Ka)	
Peak value of average current over length of stroke	10 Ka	
Duration of initial pulse (at 1/2 peak current)	60 μs	
Effective height of discharge (length of stroke)	1.5 Km	
Discharge from cloud during initial pulse	0.60 coul	
Discharge from cloud during entire stroke	20 coul	
Probability of any number of strokes per flash	0.50	1 stroke
	0.15	2
	0.10	3
	0.25	4 or more
Time per stroke in multiple strokes	.05 sec	

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While these statistics are based on limited reports and knowledge, it is intended that the values given should probably be within a ratio of 1:2 from the median values of experience.

(18) H.A.W. Notebook 95, p. 128-148.

(19) H.A.W. Notebook 99, p. 6-9.

(20) B. F. J. Schonland, "Atmospheric Electricity", Wiley; 1932/1953. (Lightning, p. 65-79; the most authoritative reference.)

(21) J. A. Chalmers, "Atmospheric Electricity", Pergamon; 1957. (Lightning, p. 235-255.)

(22) L. B. Loeb, "Thunderstorms and Lightning Strokes", Chap. 13 in "Modern Physics for the Engineer", McGraw-Hill; 1954.

(23) J. H. Hagenguth, "Theory of Lightning", Sec. 26, p. 2154-63, in "Standard Handbook for Electrical Engineers", 9 ed.; 1957. (Thunderstorm map of U.S.).

(24) A. D. Watt, E. L. Maxwell, "Characteristics of atmospheric noise", Proc. IRE, vol. 45, p. 787-794; June 1957. (Initial pulse.)

Field from lightning discharge.

To simulate the field of a single (initial) pulse of current in a lightning discharge, consider this current in an antenna of the C type, Fig. 2(a). The earth and ionosphere are assumed to be parallel planes of perfect conductivity, causing no attenuation.

h_o = effective height of lightning discharge
I_o = peak current during initial pulse, average over the height
f_o = nominal frequency of initial pulse (at which the pulse shape simulates one cycle)
$1/2f_o$ = effective duration of initial pulse, at 1/2 peak current
f_w = effective bandwidth at receiver (both sidebands)
H_r = magnetic field at receiver, peak value of RMS envelope of pulse through limited bandwidth
sub-o = lightning discharge
sub-r = receiver pulse
sub-w = bandwidth

The following formulas are obtained by multiplying (22), (23), (1) by the pulse bandwidth factor

$$\frac{f_w}{\sqrt{2}\, f_o} << 1 \tag{28}$$

This converts the peak current of the pulse to the peak value of RMS envelope of pulse current as seen by a receiver of limited bandwidth.

Spherical near field: $h_o < r < h_i < \lambda/\pi^2$

$$H_r = \frac{I_o h_o}{2\pi r^2} \frac{f_w}{\sqrt{2}\, f_o} \tag{29}$$

Cylindrical near field: $h_i < r < \lambda/\pi^2$

$$H_r = \frac{I_o h_o}{2\pi r h_i} \frac{f_w}{\sqrt{2}\, f_o} \tag{30}$$

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Cylindrical far field (radiation): $h_i < \lambda/\pi^2 < r$

$$H_r = \frac{I_o h_o}{2\sqrt{\lambda r}\, h_i} \frac{f_w}{\sqrt{2}\, f_o} \qquad (31)$$

Here we include for reference the spherical far field (radiation) in absence of ionosphere and any attenuation: $h_o < \lambda/2\pi < r$

$$H_r = \frac{I_o h_o}{\lambda r} \frac{f_w}{\sqrt{2}\, f_o} \qquad (32)$$

This may be used for comparison with computations of Watt (ref. 24).

The initial pulse of a typical lightning discharge (Watt, ref. 24) has the following values:

I_o	=	10 Ka
h_o	=	1.5 Km
$1/2f_o$	=	60 μs
f_o	=	8.3 Kc/s
$I_o h_o$	=	15 Km-Ka
$I_o/2f_o$	=	0.6 coulomb
$I_o h_o/f_o$	=	1.8 Km-coul

The last value is the moment of the discharge and its image in the ground.

(25) H.A.W. Notebook 95, p. 142-8.

(26) H.A.W. Notebook 99, p. 11.

(27) H. A. Wheeler, "The radiansphere around a small antenna", Proc. IRE, vol. 47, p. 1325-31; Aug. 1959.

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References on heating of wires.

The following references provide some information relating to the temperature rise in a conductor, as may be encountered in a high-current sending antenna. They are listed chronologically.

This subject is one that has inherited a confusion of units, and the forward-looking MKS units (used with Kelvin scale of temperature) are seldom even included in conversion tables. The CGS units are most common, as denoted by asterisk (*) in the list.

(28)* F. E. Fowle, "Smithsonian Physical Tables", 8 ed.; 1934. (Thermal conductivity tables, p. 272-279.)

(29) A. E. Knowlton, "Standard Handbook for Electrical Engineers", 7 ed., McGraw-Hill; 1941. (Temperature rise of horizontal wire in still air, conditions indefinite, good curves, Sec. 4-287.)

(30) A. I. Brown et al, "Introduction to Heat Transfer", McGraw-Hill; 1942. (Thermal conductivity table, p. 17-8. Wire in air, p. 117-8.)

(31)* F. E. Terman, "Radio Engineers' Handbook", McGraw-Hill; 1943. (Solid dielectrics, thermal conductivity, table p. 111. Taken from General Radio Experimenter, June, 1939.)

(32)* L. R. Ingersoll et al, "Heat Conduction", McGraw-Hill; 1948. (Thermal conductivity tables, p. 241-5.)

(33)* H. Pender et al, "Electrical Engineers' Handbook - Electric Power", 4 ed., Wiley; 1949. (Thermal conductivity table, p. 2-1-2. Current capacity of wire in air, p. 14-185. Current capacity, various cases, p. 14-210-27.)

(34) W. H. McAdams, "Heat Transmission", McGraw-Hill; 1954. (Theory of convection, horizontal wire in air, p. 176. Wire in air flow, p. 259. Conversion factors, except MKS, p. 444. Thermal conductivity tables, p. 446-60.)

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(35)* A. E. Knowlton, "Standard Handbook for Electrical Engineers", 8 ed., McGraw-Hill; 1957. (Bibliography on current capacity of wire in air, Sec. 4-144. Thermal insulating materials, conductivity table, Sec. 4-607. Current capacity of insulated wire in air, Sec. 15-30.)

(36)* C. D. Hodgman, "Handbook of Chemistry and Physics", 40 ed., Chemical Rubber Publ. Co.; 1958. (Heat conductivity tables, p. 2431-40.)

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Heat conductivity of various materials.

The following table includes materials that may be involved in computing the heating of wires in a high-current antenna. The values given are effective at room temperature (about 20° C) except for ice and snow.

Material	CGS	MKS	Reference
copper	0.92	396	(36)
aluminum	0.49	205	(36)
air	.000058	.024	(36)
water	.00143	0.60	(36)
ice	.0053	2.2	(32)
snow, fresh (about 1/8 density of ice)	.00026	0.11	(28)
snow, old (about 1/2 density of ice)	.0012	0.50	(28)
quartz (2 axes)	.030	12.6	(36)
	.016	6.7	(36)
polystyrene	.0004	0.17	(31)
paraffin	.0006	0.25	(36)
rubber, hard	.00038	0.16	(35)
rubber, para	.00045	0.19	(36)
rubber compounds	.00048	0.20	(33)
earth's crust (average)	.004	1.7	(36)
quartz sand, dry	.00063	0.26	(32)
quartz sand, 8% moisture	.0014	0.59	(32)
sandy clay, 15% moisture	.0022	0.92	(32)
wet soils (range)	.003	1.3	(32)
	.008	3.4	(32)
wet mud	.002	0.84	(32)
concrete, average stone	.0022	0.92	(32)
concrete, dams	.0058	2.4	(32)

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1 calorie	=	4.187 joules
CGS unit of conductivity	=	$\frac{\text{calorie}}{\text{sec x cm x }^{o}\text{C}}$
MKS unit of conductivity	=	$\frac{\text{watt}}{\text{meter x }^{o}\text{C}}$
1 CGS unit	=	418.7 MKS units

It is noted that the so-called CGS unit is inconsistent with the CGS system in several respects. It is "rationalized", being defined in terms of rectangular coordinates, whereas the CGS system is "unrationalized", being defined in terms of spherical coordinates. Also it is defined in terms of the calorie instead of the erg. The temperature difference scale (Centigrade or Kelvin) is beyond the CGS system, but is generally accepted for scientific purposes.

The MKS unit is consistent with the prevalent MKS system. It is likewise "rationalized". It uses the joule, not the calorie. The temperature-difference scale (Centigrade or Kelvin) is beyond the MKS system, but is commonly used therewith.

(37) H.A.W. Notebook 99, p. 17-20.

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Field from lightning discharge (cont.).

The actual discharge of a lightning stroke has an initial pulse of current followed by a trailing edge which decays slowly. It appears that the initial pulse accounts for only a small fraction (about .03) of the entire discharge (see p. 34). Therefore the trailing edge is the major contributor to radiation at frequencies much below the spectral peak (about 8 Kc).

This effect is shown in a recent experimental report (Ref. 39). For a short pulse, at frequencies below the peak, the spectral coefficient of amplitude would decrease with the first power of the frequency. Instead, this report shows it decreasing approximately with the 1/2 power, down to 1 Kc (the lower limit of the observations).

In the absence of experimental evidence, it is conjectured that this 1/2-power variation continues down to about 10 c/s, as well it may without violating any obvious rules. At still lower frequencies, the entire discharge would behave as a short pulse, so a first-power variation would be approached asymptotically. It is conceivable that the 1/2-power variation may have a theoretical basis, similar to the discharge of a line with uniformly distributed series resistance and shunt capacitance.

The preceding formulas (29), (30), (31) are modified for the trailing edge by applying the factor

$$\sqrt{f_o/f} \qquad (33)$$

No rules are violated if this factor complies with the conditions

$$1 < \sqrt{f_o/f} < \frac{\text{entire pulse}}{\text{initial pulse}} \qquad (34)$$

In the present situation, this is applicable over the frequency range of about 7 c/s to 8 Kc/s; this happens to include the entire range of present interest. The following formulas give the peak value of the envelope of RMS pulse current, as seen by a receiver of limited bandwidth ($f_w << f$).

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Spherical near field: $h_o < r < h_1 < \lambda/\pi^2$

$$H_r = \frac{I_o h_o}{2\pi r^2} \frac{f_w}{\sqrt{2f_o f}} \tag{35}$$

Cylindrical near field: $h_1 < r < \lambda/\pi^2$

$$H_r = \frac{I_o h_o}{2\pi r h_1} \frac{f_w}{\sqrt{2f_o f}} \tag{36}$$

Cylindrical far field (radiation): $(2/\pi^2)\, h_1 < \lambda/\pi^2 < r$

$$H_r = \frac{I_o h_o}{2h_1\sqrt{r\lambda}} \frac{f_w}{\sqrt{2f_o f}} = \frac{I_o h_o f_w}{2h_1\sqrt{2rv_c f_o}} \tag{37}$$

in which $v_c = f\lambda$ = wave velocity . The latter form shows that this value is independent of the operating frequency (f).

If the bandwidth (f_w) is so narrow that successive pulses are widened enough to overlap, the envelope of the pulses will increase. The nominal pulse width is $1/f_w$. If the average frequency of the pulses is $f_p > f_w$, the RMS value of the overlapping pulses is obtained from the preceding formulas by applying the following factor:

$$\sqrt{f_p/f_w} > 1 \tag{38}$$

The cylindrical far field becomes:

$$H_r = \frac{I_o h_o}{2h_1}\sqrt{\frac{f_w f_p}{2rv_c f_o}} \qquad \text{(independent of } f\text{)} \tag{39}$$

(38) H.A.W. Notebook 99, p. 27-31.

(39) W. L. Taylor, A. G. Jean, "VLF radiation spectra of lightning discharges", Jour. of Res. of NBS, vol. 63D, p. 199-204; Sept.-Oct. 1959.

Note: Some workers call the trailing edge, the "slow tail".

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Noise level from distant lightning discharges.

It is desired to estimate the prevalent noise levels off the coast of Maine, for purposes of experiments rather than communication. For this objective, the following pattern is suggested to represent areas of lightning (thunderstorms) that may give typical noise levels.

The world maps (Refs. 40, 41) show that the greatest concentration of lightning occurs in three tropical areas, namely, America, Africa and Indonesia. In summer, the Gulf of Mexico is second to none; in winter, the corresponding area moves south to Brazil, and there is no nearer area of concentration.

Since the three principal areas are in different time zones, and storms are most frequent in the latter part of the day, only one area is most likely to be active at one time. Therefore, at any particular time, we may assume that much of the world's lightning is occurring at one of these three areas.

The U.S. map (Ref. 42) shows the concentration of thunderstorms toward the Gulf area. In terms of the average number of thunderstorm days per year, we note the following:

Florida	80/365	=	0.22
Mid-U.S.	50		0.14
Maine	20		0.055
West coast	5		0.014

On perhaps 3/4 of the days, there is little lightning in the east half of U.S., and possibly in all of U.S.

During 2/3 of every day, there is seldom a lightning area closer than Africa or Indonesia. The corresponding low noise level may be estimated by assuming the world's lightning to be occurring in one of those areas. During about half of the days in summer, there may be another part of the day, in which this activity is concentrated in the Gulf area. Any closer activity is infrequent in days and transient in hours. It appears that the lightning activity in the Gulf or in any closer areas can be avoided in making experiments.

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The distances from Maine to the further sources are:

Equatorial Africa	1/4 great circle
Indonesia	3/8 (over N. Pole)

Since the latter is approaching the antipode, there is appreciable focusing action of the earth's curvature. Therefore computations will be based on 1/4 great circle, which is 10,000 Km. Attenuation will be ignored, but all paths longer than the direct path will be assumed to be attenuated sufficiently to avoid substantial contribution to the total noise.

The following example should give a rough idea of the median RMS noise (H_r) in Maine during the quieter 2/3 of every day. See (39) on p. 43; also see p. 34, 37.

I_o = 10 Ka
h_o = 1.5 Km
h_1 = 80 Km
r = 10,000 Km
v_c = 300,000 Km/sec
f_o = 8.3 Kc/s
f_w = 1 c/s
f_p = 100 c/s
H_r = 0.13 μa/m

This value is independent of the operating frequency (f) under the specified limitations, which are likely to be valid at 30-300 c/s. The actual value may be considerably less, if there is substantial attenuation.

In the neighborhood of 100 c/s, there is fair agreement between this example and some observations of Willis in England. His report will be analyzed in a later section.

(40) W. Q. Crichlow et al, "Worldwide Radio Noise Levels Expected in the Frequency Band 10 Kc to 100 Mc", NBS, C-557; Aug. 25, 1955.

(41) W. Q. Crichlow, "Noise investigation at VLF by NBS", Proc. IRE, vol. 45, p. 778-782; June 1957. (World map and curves of noise level, summer night.)

(42) J. H. Hagenguth, "Theory of lightning", Standard Handbook for Elec. Engrs., 8 ed., McGraw-Hill, p. 2154-63; 1957. (U. S. map of thunderstorm days per year.)

(43) H.A.W. Notebook 99, p. 38-56.

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AF magnetic fluctuations.

H. F. Willis has observed and reported fluctuations of horizontal magnetic field at frequencies of 5-800 c/s, observed at an isolated site in England (Ref. 44). He reports omnidirectivity in azimuth and does not indicate any difference between April and November. His report is in terms of "typical values for the fluctuations", which we may take to be comparable with the RMS values. They were observed on a bandwidth of 4 c/s, which is narrow enough to assure a random noise of overlapping pulses. They are reduced to a bandwidth of 1 c/s.

His observations can be summarized by two numbers, as follows:

At 100 c/s, $H = 10^{-9}$ gauss (oersted) $= .08\ \mu a/m$

At frequencies of 5-800 c/s, $H \propto f^{-3/2}$

This value at 100 c/s is comparable with the preceding example for Maine (0.13 μa/m). On the other hand, the writer's theory gives a value that is nominally independent of frequency, which differs from the Willis report. His observations seem to offer a good starting point for further theoretical and experimental studies.

His report of omnidirectivity is contrary to expectations (and contrary to experience at VLF). There is a question how literally this report should be taken. Perhaps the magnetic field merely occurred in all directions at different times.

The amplitude decreasing with increasing frequency could be explained in several ways. There is increasing attenuation in propagation, and a reasonable amount could cause the observed trend.

Conversely, the amplitude increasing at lower frequencies may be caused by a relatively small number of unusually large discharges (single or multiple) which disproportionately contribute to the total energy. Even in typical discharges, the "trailing edge" or "slow tail" may contribute to this trend more than has been contemplated. It is even possible that the upward trend at the lowest frequencies has a principal cause other than lightning.

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(44) H. F. Willis, "Audio-frequency magnetic fluctuations", Nature, vol. 161, p. 887-8; June 5, 1948.

(45) H.A.W. Notebook 99, p. 48, 56.

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Radiation from a wire on the ground.

If a long wire is located on the ground, with its ends connected to ground electrodes, it radiates like a vertical loop with return circuit at some depth in the ground. It is assumed that the wire is insulated, if necessary to assure that nearly all the current returns through the ground electrodes. The current may be provided from a generator connected in series with the wire. (Reference is made to Fig. 2(b), showing a complete loop above ground.) In the case of a long wire on the ground:

a	=	radius of wire
g	=	length of wire between ground electrodes
h_m	=	effective height of the wire, relative to return path in the ground
δ	=	skin depth in the ground
R_c	=	377 ohms = wave resistance of free space
R_g	=	ground return resistance between ideal plane electrodes
X_g	=	reactance of wire and ground return
R_h, X_h	=	extra resistance and reactance of practical electrode at each end
p_g	=	R_g/X_g = power factor

The significant properties of this loop radiator are its effective area and its ground resistance, since most of the power is dissipated in the ground. The reactance is significant relative to tuning and bandwidth.

The following restrictions enable the simplest analysis:

$$a \ll \delta \ll g < \lambda/2\pi$$

From the viewpoint of a receiving antenna, it can be shown that the voltage induced in this wire has the same magnitude as would be induced in a loop of a certain area just above a perfect ground plane. To express this area, we take the length of the wire (g) multiplied by the following effective height:

$$h_m = \delta/\sqrt{8} \tag{40}$$

Then the effective area is gh_m , and this can be used in any formula for a sending or receiving loop. This effective height is equal to the average depth of all parts of the return current in the ground, taking only their components that are in phase with the total current. Naturally the effective height is somewhat less than the skin depth.

The resistance of the ground return is

$$R_g = \frac{1}{8}\omega\mu_o g = \frac{1}{8}R_c\frac{2\pi g}{\lambda} = 47.1\frac{2\pi g}{\lambda} = 296\ g/\lambda \tag{41}$$

This value is obtained from Carson's derivation, for the limiting case of a thin wire on the ground. It is independent of the conductivity and skin depth. It happens to be the resistance of a simple conductor having the conductivity of the ground and a cross section of $4\delta \times \delta$, if it carried uniform current density.

The reactance of this wire, with this ground return, is expressed relative to the resistance:

$$X_g/R_g = 1/p_g = \frac{4}{\pi}\left[\ln\frac{\sqrt{2}\ \delta}{a} + \frac{1}{2} - 0.577\right] = \frac{4}{\pi}\ln\frac{1.31\ \delta}{a} \tag{42}$$

This is equal to the reactance the wire would have if it were located above a perfect ground plane at a height of $0.655\ \delta$.

The above formulas do not include the internal resistance and reactance of the wire. If the skin effect in the wire is negligible, its resistance has the DC value; also its internal reactance may be included in (42) by adding 1/4 to the logarithm.

The extra resistance ($2R_h$) of the end electrodes should be made somewhat less than the ground return resistance (R_g). If the same size wire, but not insulated, were inserted vertically in the ground, to a depth exceeding the skin depth, its series resistance and reactance would be approximately:

$$R_h = X_h = \frac{1}{4\pi}R_c\frac{2\pi\delta}{\lambda}\ln\frac{0.8\ \delta}{a} \tag{43}$$

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If the same wire were buried just below the surface, its resistance would be greater but not twice as great. The use of N radial wires would reduce the resistance, but the factor would not be as small as 1/N .

It is interesting to compare an insulated wire on the ground (Ref. 46) with one buried much deeper than the skin depth (Ref. 47). Each is connected between ground electrodes at the ends. In these two cases, the ground return resistance is the same. However, the buried wire has slightly less reactance, to the extent of removing the 1/2 in the brackets in formula (42).

The utility of a wire on the ground resides in the ease of supporting it over a long distance. If the ground conductivity is as low as 1 millimho/meter, the skin depth at 100 c/s is 1.6 Km and the effective height is 560 meters. This effective height could be utilized for a great length, requiring no supporting towers. There is a problem, finding low conductivity to a sufficient depth over a large area.

(46) J. R. Carson, "Wave propagation in overhead wires with ground return", BSTJ, vol. 5, p. 539-54; Oct. 1926. (Called to my attention by C. A. Martin:)

(47) R. K. Moore, "The theory of radio communication between submerged submarines", Cornell Univ., thesis; June 1951. (R and X of submerged insulated wire between ground electrodes.)

(48) H.A.W. Notebook 99, p. 52-5.

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Report 914

UNDERGROUND LF ANTENNAS

By Harold A. Wheeler

To Developmental Engineering Corporation

1959 SEP 28 Job 401

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Contents

This is a cumulative report for collecting information developed in the course of studies and consultation relating to medium-range communication between underground LF antennas. The following list will be brought up to date and reissued with each release of additional pages.

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Loop antennas for 250 Kc.

A system is proposed on the basis of these conditions.

Distance	r = 400 Km = 250 mi
Frequency	f = 250 Kc
Wavelength	λ = 1.2 Km
Depth to center of each loop	d = 15 m = 50 ft
Conductivity of ground	σ = .01 mho/meter
Skin depth in ground	δ = 10 m
Depth attenuation of magnetic field	$\exp -d/\delta$ = 0.223 = 1.5 nap = 13.0 db

The frequency is chosen as a compromise between maximum efficiency and maximum reliability. Selective fading is to be expected, with a path difference of about 25 λ or more. To overcome this handicap, we may use FSK with frequency shift of about 1 % or 2.5 Kc, with a receiver having two narrow channels, and differentially combine the rectified signals. This will reliably give normal reception with path differences up to 50 λ, to be expected at 200 Km. No selective fading is expected at shorter distances.

For omnidirective sending and receiving, each end has a pair of crossed vertical loops coupled in phase quadrature. The sender would have separate power amplifiers; the receiver might have separate preamplifiers. Or the receiver could have separate amplifiers and detectors, then combine the output, thereby avoiding the quadrature requirement in favor of the diversity principle.

The sender loop may be made of one turn of 4 parallel conductors, forming a 10-meter square with mean depth of 15 meters. Each conductor is aluminum tubing of 2 inches outside diameter, and the four in parallel are spaced on the corners of square insulating frames of 2-ft diagonal. They may be located in a tunnel lined with tile or concrete (no metal). The parallel conductors are arranged to divide the current equally, as by twisting one turn along each side of the square loop. The following are the conditions at the sender.

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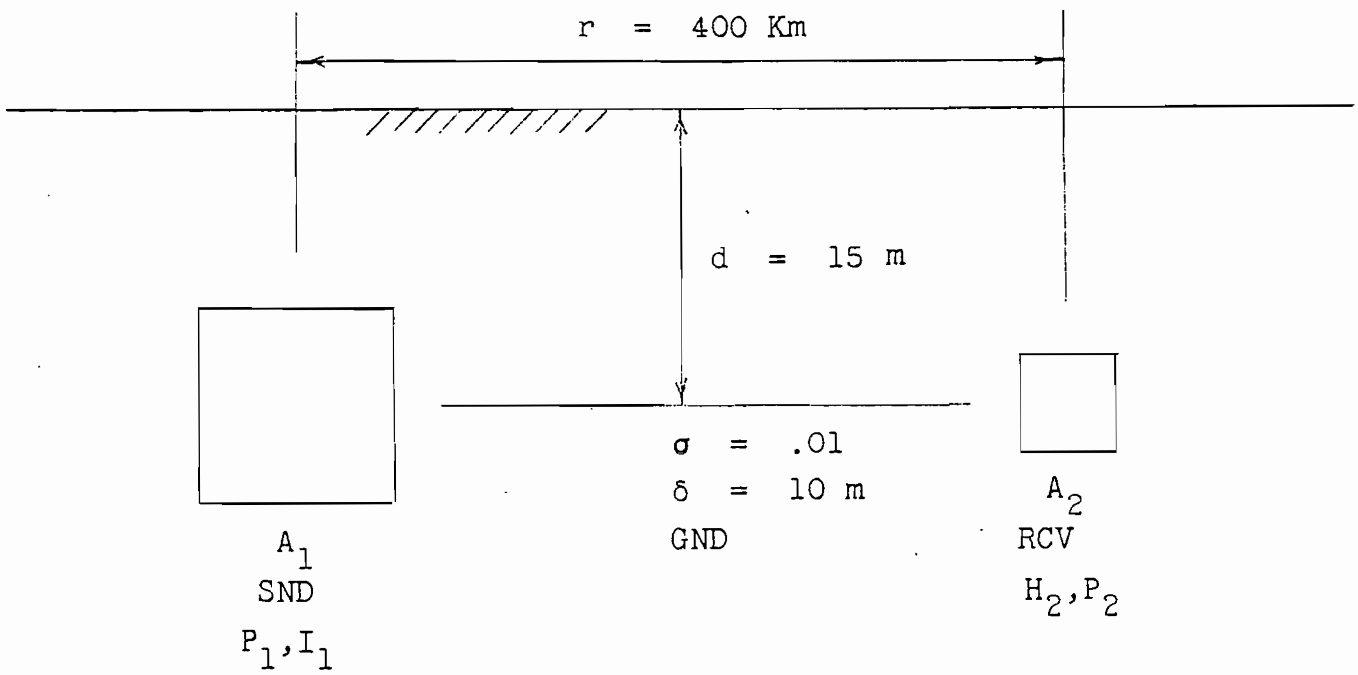

Fig. 1 - Underground antennas for sending and receiving.

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Loop area	A_1	= 100 m^2
Inductance	L_1	= 25 μh
Capacitance to tune	C_1	= .016 μh
Reactance	X_1	= 39 ohms
Series resistance (caused by ground conduction)	R_1	= 2.25 ohms
Parallel resistance (equivalent to series resistance)	R_1'	= 680 ohms
Power factor	p_1	= R_1/X_1 = .058
Power	P_1	= 50 Kw
Current	I_1	= 150 amp RMS
Voltage	V_1	= 5.8 Kv RMS

Effective current in loop (the current which, in an equal loop above ground, would radiate the same power as the actual current in the underground loop)

	I_{10}	= 33.5 amp
Radiation resistance above ground	R_{10}	= 0.30 milohm
Radiation power	P_{10}	= 0.34 watt
Radiation efficiency	P_{10}/P_1	= 6.8 microns
		= -51.6 db

(1 micron = 1 mil^2 = 10^{-6})

The loss of radiation efficiency is made of two parts, the ground-conduction loss relative to free space radiation (38.6 db) and the depth attenuation (13.0 db).

From the effective current in an equal loop above ground, we compute the field above ground at the receiver:

Magnetic field	H_{20}	= .036 μa/m
Electric field	E_{20}	= 14 μv/m
Available power (from a vertical loop)	P_{20}	= .045 μw

This must be compared with the noise level expressed in the same terms. The thermal noise, based on the bandwidth that is here proposed, is as follows.

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Bandwidth	Δf	= 1 cycle/second
Temperature	T	= 300° K
Available power	P_n	= $kT\Delta f$ = 4.14×10^{-21} watt
Magnetic field	H_n	= .0113 $\mu\mu$a/m
Electric field	E_n	= 4.25 $\mu\mu$v/m
Signal/thermal noise	P_{20}/P_n	= 11×10^{12} = 130.4 db

Here we insert a correction for curved earth (3 db) and ground loss (9 db) which reduces this last ratio to about 117 db.

Atmospheric radio noise in U.S., during its greatest activity in summer nights, has a median value about 110 db above thermal noise. The above signal covers this value by about 7 db, which may possibly be sufficient for reliable operation with FSK. At 4/5 distance (320 Km = 200 mi), the margin is 7 db greater, or 14 db, which is probably adequate for reliable operation.

The receiver requires underground crossed loops sufficiently large to assure that the atmospheric noise is somewhat greater than the thermal noise. Since this ratio above ground is about 117 db, the receiver loop should have a radiation efficiency substantially greater than -117 db; this result is easily obtained with a moderate size of loop. Still greater efficiency (by 20 db or so) would take full advantage of the periods of much lower atmospheric noise (as in winter days).

The design of each receiver loop is based on one wide turn (current sheet) on a meter-cube form in a 2.2-meter (7-ft) cubic room with concrete walls (free of any metal reinforcement). These cubic forms are approximately equivalent to spherical coil and radome of radii 0.7 m and 1.4 m .

Underground magnetic field	H_2 = .0080 μa/m
Induced voltage	V_2 = .016 μv
Inductance	L_2 = 0.62 μh
Reactance	X_2 = 0.97 ohm
Capacitance to tune	C_2 = 0.65 μf
Power factor (caused by ground conduction)	p_2 = .00245 = 2.45 mils
Series resistance	R_2 = pX = 2.4 milohms
Parallel resistance (equivalent to series resistance)	R_2' = 400 ohms
Resonant voltage	$V_2' = V_2/p_2$ = 6.5 μv
Radiation efficiency	-66.0 db

A practical design might obtain a voltage step-up ratio of 35 by making the loop of 7 turns, and coupling to the tuning capacitor through a balanced-to-unbalanced transformer with a turns ratio of 1:5 on a closed ferrite core. The conditions on the secondary would then be as follows.

Effective number of turns	n = 35
Capacitance to tune	C/n^2 = 530 μμf
Resonant voltage	nV_2' = 230 μv
Parallel resistance	n^2R_2' = 0.49 megohm

Each of the crossed loops may be made of 7 turns of one-inch diameter thin-wall aluminum tubing, flattened locally for bending at the corners.

The receiver antenna has a large factor of safety for the specified requirements. The low value of radiation efficiency loses only 66 db out of the 110 db by which the highest median atmospheric noise exceeds the thermal noise; part of the remaining 44 db may be sacrificed to some practical advantages in design, as follows. Increasing the series resistance by 8 times (to give a 3-db bandwidth of 20 mils or 5 Kc) would sacrifice 9 db. Doubling the depth (to 30 meters) would increase the attenuation by 13 db. The amplifier might introduce excess noise of 3 db. The total of these changes would increase the receiver excess noise (relative to thermal radiation above ground) from 66 to 91 db, which is still 19 db less than the highest median atmospheric noise. In any case, the theoretical performance might be degraded by about 3 to 6 db by practical limitations.

The most serious difficulty is the radiation of power above ground from a small sending antenna located underground. In the above example, there is a loss of 51.6 db. The result is marginal for overcoming the highest median atmospheric noise at the receiver. Increasing the radiation power may be accomplished by any of the following changes; conversely, any particular requirement may be relaxed at the expense of another.

1. Increasing the power to the sender loop.
2. Decreasing the depth to the loop center.
3. Increasing the size of the loop (especially the width rather than the height).
4. Decreasing the ground conductivity (around and above the loop).
5. Decreasing the bandwidth (now 1 cycle/second).
6. Employing a large horizontal array of loops (to concentrate the radiation at low angles).
7. Complicating the type of modulation and detection.

(1) H. A. Wheeler, "Radio wave propagation formulas", Hazeltine Report 1301WR; May 11, 1942, revised June 1945. (Interception area of dipole in free space.)

(2) F. E. Terman, "Radio Engineers Handbook", McGraw-Hill, 1943. (Propagation over spherical earth, curves including 300 Kc and dependence on ground conductivity, p. 681-3. Ground constants, p. 709. Low-frequency propagation, p. 733-6.)

(3) J. A. Pierce, "Propagation", Rad. Lab. Series, vol. 4, chap. 5, p. 121-169; 1948. (Includes Loran experience and curves for 180 Kc; ground wave, sky wave, selective fading, path difference, required signal at receiver.)

(4) W. Q. Crichlow, "Noise investigation at VLF by the National Bureau of Standards", Proc. IRE, vol. 45, p. 778-82; June 1957. (Charts of atmospheric radio noise from 10 Kc to 30 Mc, for summer night, p. 781.)

(5) H. A. Wheeler, "Fundamental limitations of a small VLF antenna for submarines", IRE Trans., vol. AP-6, p. 123-5; Jan. 1958. (A spherical coil in a spherical radome.) Formulas (3) and (4) contain errors.

(6) H. A. Wheeler, "The spherical coil as an inductor, shield, or antenna", Proc. IRE, vol. 46, p. 1595-1602; Sept. 1958. (Radiation power factor in free space or under water.) Formulas (29) and (31) contain errors.

(7) H. A. Wheeler, "The radiansphere around a small antenna", Proc. IRE, vol. 47, p. 1325-31; Aug. 1959.

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Radiation efficiency of underground loop.

(It is noted that the formulas in this section may not give exactly the numerical values in the preceding section, since that was compiled before some of these formulas were completed and checked.)

A small loop antenna is submerged in a conductive medium. The largest dimension of the loop is much less than the skin depth in the medium at the operating frequency. The depth of submersion is much greater than the largest dimension of the loop. The medium has a horizontal plane boundary, above which there is free space (air).

The radiation efficiency of this loop is the power ratio of the radiation into the free space (above ground) over the input power, most of the latter being dissipated in the ground by the currents induced in its conductance. This can be computed simply for an air-core spherical coil, as follows (see reference 6).

Frequency	f	(cycles/second)	
Wavelength	λ	(meters)	
Conductivity in ground	σ	(mhos/meter)	
Skin depth in ground	δ	$= \sqrt{\lambda/\pi\sigma R_o} = \frac{1}{2\pi}\sqrt{\lambda/30\sigma}$ (meters)	
Depth of center of coil	d	(meters)	
Wave resistance in free space	R_o	= 377 ohms	
Radius of coil	a	(meters)	
Radius of outer radome	a'	(meters)	
Number of turns	n		
Effective area	A	$= \frac{2}{3}\pi a^2 n$ (meter^2)	(1)
Inductance	L	$= \frac{2\pi}{9}\mu_o a n^2$ (henries)	(2)
Reactance	X	$= \frac{2\pi}{9} R_o \frac{2\pi a}{\lambda} n^2$ (ohms)	(3)

Radiation power factor of this coil just above ground

$$p_o = \frac{2}{3}\left(\frac{2\pi a}{\lambda}\right)^3 \qquad (4)$$

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Corresponding radiation resistance

$$R_{ao} = p_o X \tag{5}$$

Dissipation power factor of this coil in a radome in the ground $(a < a' \ll d)$

$$p' = \frac{2a^3}{3a'\delta^2} \tag{6}$$

Corresponding dissipation resistance and equivalent parallel conductance

$$R'_a = p' X \tag{7}$$

$$G'_a = \frac{3\sigma a^2}{2\pi a'} \tag{8}$$

Radiation efficiency for this case

$$(p_o/p') \exp{-2d/\delta} = a'\delta^2 (2\pi/\lambda)^3 \exp{-2d/\delta} \ll 1 \tag{9}$$ $\propto f^2$

It is noted that the conductivity and frequency are implicit in δ^2 which is proportional to λ/σ. In this case, the radiation efficiency is independent of the coil size but is proportional to the radome radius; if the coil is substantially smaller than the radome, the radiation efficiency is also independent of the coil shape.

The next case is based on a spherical coil immersed in the medium without a radome, so the ground conductivity is effective in all space outside and inside. (The dissipation inside is 1/5 as much as outside.)

Dissipation power factor of this coil in the ground

$$p = \frac{4a^2}{5\delta^2} \tag{10}$$

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Corresponding dissipation resistance and equivalent parallel conductance

$$R_a = p\,X \tag{11}$$

$$G_a = \frac{9}{5\pi}\,\sigma a \tag{12}$$

Radiation efficiency for this case

$$(p_o/p)\,\exp{-2d/\delta} = \frac{5}{6}\,a\delta^2\,(2\pi/\lambda)^3\,\exp{-2d/\delta} \ll 1 \tag{13}$$

Except for the exponential factor, this would be proportional to $a/\sigma\lambda^2$.

In the latter case, ~~as compared with the spherical coil~~, a more concentrated coil of the same radius would give~~s~~ a somewhat higher efficiency, probably less than twice as great. Therefore the above formulas place a lower limit on the performance of some practical shapes.

In these formulas, conductor resistance in the coil is ignored, also the electric-field loss in the nearby ground.

If the dimensions of the loop become comparable with the skin depth, the radiation efficiency is greater than that given by the above formulas. ?

(5) (6) (7) References (p. 9) for spherical coil, radome, underwater location.

NB 93, p 65-8.

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Report 967

VLF TRANSMITTER NOTES

By Harold A. Wheeler

To Continental Electronics Mfg. Co.

1960 SEP 6 Job 404

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Contents

This is a cumulative report for collecting information developed in the course of studies and consultation relating to the VLF transmitter at Cutler, Maine. The following list will be brought up to date and reissued with each release of additional pages.

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Ammeter utilizing air-core toroid.

A high-current RF ammeter is needed to measure the current in each half of the antenna, while the power is being delivered to both halves operating together or to either half operating alone. The nominal highest current values are as follows:

Each half, both operating:	1800 amp RMS
Either half, operating alone:	2800 amp RMS

It is conjectured that the latter value might go as high as 4000 amp, so this is taken as the maximum value to be measured.

The ammeter is to be separated from the high-current bus by an airgap sufficient for insulation against 15 Kv RMS, and is to be shielded from the bus by a metal casting.

The frequency range is 14-30 Kc, with highest current at 14 Kc and perhaps half this current at 30 Kc. The design is more difficult for lower frequencies, so it is based on the above current values at 14 Kc.

The arrangement here proposed is a toroidal coupling around the high-current bus, followed by a series inductor and a thermocouple of intermediate current rating. The DC output of the thermocouple goes through a filter to remove any RF currents, then through a long line (3000 ft) to a zero-current electromechanical millivoltmeter (Weston). The latter is independent of the series resistance of filter and line, and any number can be connected in parallel without interaction.

Fig. 1 shows the essential dimensions of the shield which forms the boundary of the toroidal air core to be used for coupling with the high-current bus (shown in the center). It may be an aluminum casting in 2 or 4 parts. It forms a conductive enclosure which is closed except for the gap around the outer edge. It behaves as a Faraday shield, which excludes electric field but admits magnetic field.

The essential partition is in the plane A-A, separating the casting into upper and lower halves. The faces are machined. The two halves are clamped together, in close contact at the inner circle, but separated by a thin layer of dielectric at the outer circle (say .005"

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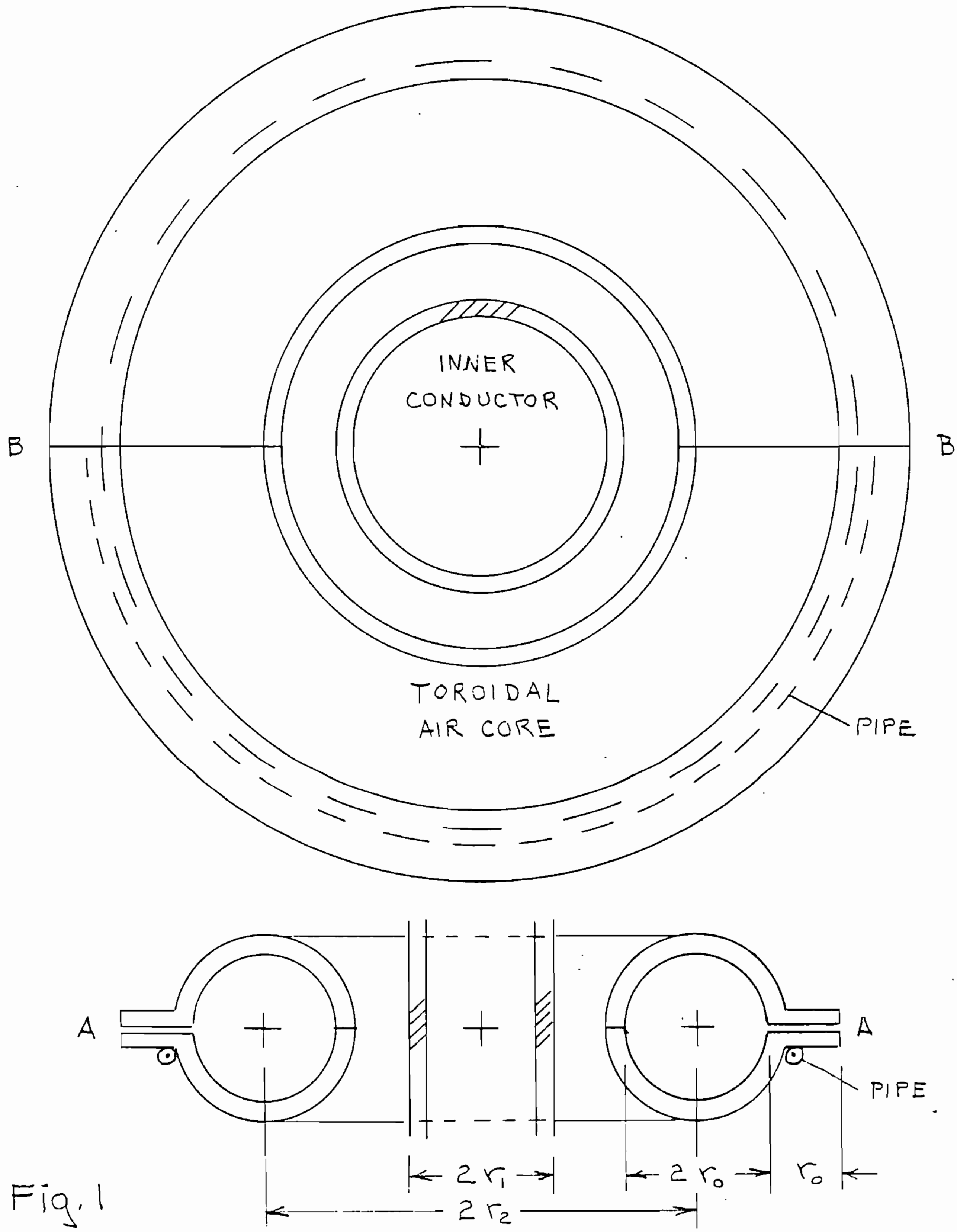
INNER
CONDUCTOR
B
B
TOROIDAL
AIR CORE
PIPE
A
A
PIPE
$2r_1$
$2r_o$
r_o
$2r_2$

Fig. 1

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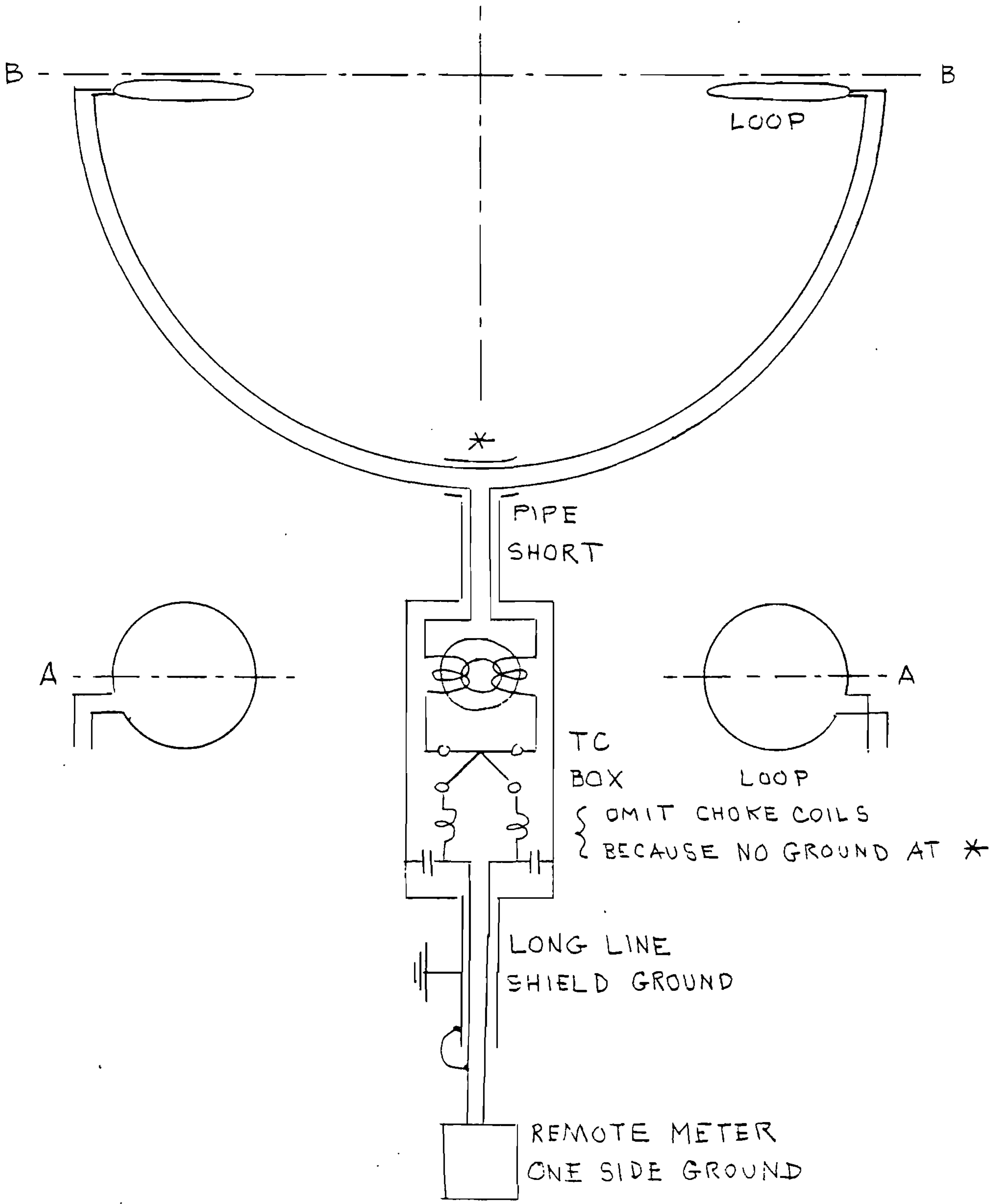
B
B
LOOP
*
PIPE
SHORT
A
A
TC
BOX
LOOP
OMIT CHOKE COILS
BECAUSE NO GROUND AT *
LONG LINE
SHIELD GROUND
REMOTE METER
ONE SIDE GROUND

Fig. 2

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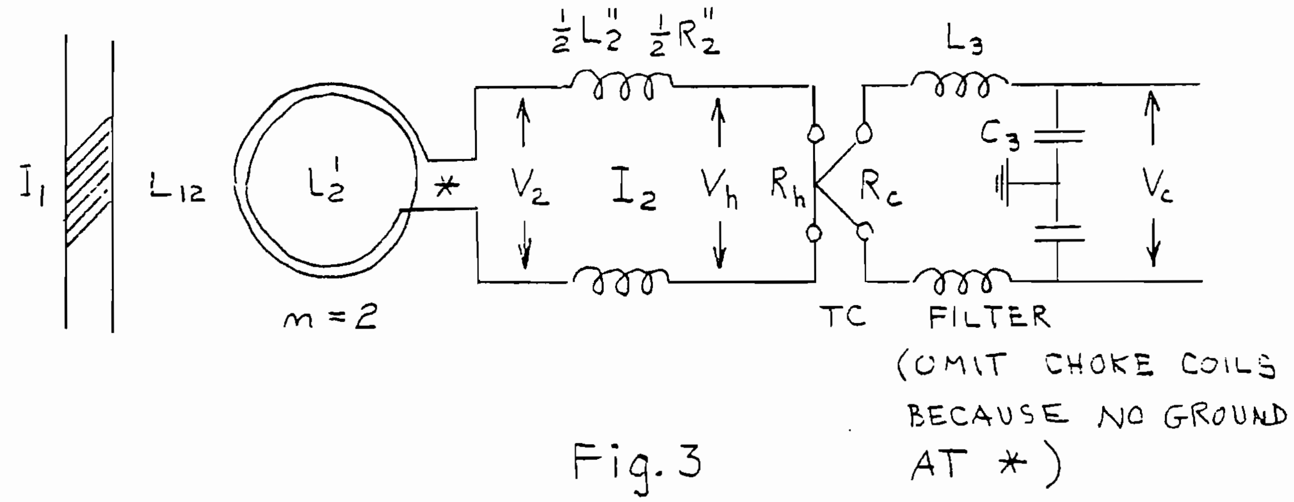
$\frac{1}{2}L_2''$ $\frac{1}{2}R_2''$
L_3
I_1
L_{12}
L_2'
*
V_2
I_2
V_h
R_h
R_c
C_3
V_c
m = 2
TC
FILTER
(OMIT CHOKE COILS
BECAUSE NO GROUND
AT *)

Fig. 3

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thickness of mylar). This dielectric should separate the faces by an amount much less than the skin depth in the casting (.020" in Al at 30 Kc).

The second partition, in the plane B-B, may be utilized to enable the shield to be removed from around the bus. Alternatively, a section of the bus may be removable. The coupling circuits may all be mounted on the lower front part of the shield, not crossing any planes of partition.

The small pipe shown on the lower half is located around the front semicircle only, and is bonded to the shield along its entire length. It will be used to shield the leads of the coupling loops to be located in the shield. It provides both electric and magnetic shielding, as well as mechanical protection. Use Cu or Al tubing.

Fig. 2 shows the arrangement of two single-turn coupling loops to be located on opposite sides in the toroidal shield. This circuit is mounted on the lower front part of the shield. Each loop is self-supporting and insulated, and is large enough to fit just inside the toroid shield, enclosing nearly all of the magnetic flux in the toroid. The pair of loops are connected in series to form a two-turn coupling.

By locating the two loops on opposite sides, two sources of error are canceled out. One is the leakage of ambient "cross field" into the shield. The other is the first-order effect of any eccentricity of the shield on the straight center conductor (or the multiple conductor to be used for calibration). Both of these effects are reduced to about 1/10 of their "free-space" values by the use of a very small gap between wide flanges around the outer edge.

Fig. 3 shows the essential circuit arrangement. The toroidal shield and coupling loops (2 turns) provide mutual inductance L_{12}. The current in the thermocouple heater R_h is then determined by the total series inductance L_2. The current ratio is

$$I_2/I_1 = L_{12}/L_2 \ll 1 \qquad (1)$$

As in the case of any meter transformer, the circuits are designed to make this ratio independent of frequency and incidental resistance.

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Examples of dimensions and circuit values will be given, together with precautions for stabilizing the current ratio.

If the thermocouple is in contact with its heater, which is customary, the output circuit may require the series choke coils and shunt bypass capacitors for RF isolation between the thermocouple and the output leads. (See remarks below relative to chokes and grounding.)

Fig. 2 shows the shielding from the toroid to the long line. The shield pipe forms the only external ground connection. The thermocouple box may be separated from the toroid by as little as a few inches or as much as a few feet, this connection being included in the calibration. This box should not be too closely adjacent to the toroid. Any circuit grounding is made to the inside of the shielding.

The plan shown is based on a single circuit ground at the output, where the AC-power amplifier circuit presumably has one side grounded. In this case, the choke coils L_3 should be omitted. On the other hand, if the circuit has an ungrounded output meter, the single ground may be located at the inter-loop connection (*); then the choke coils are needed.

The thermocouple junction circuit should be fused at the output terminals to protect against grounding faults. This will also protect the heater, which has greater current capacity.

Referring to the inductors $\frac{1}{2}L_2''$, they may be wound on the same core and measured in series, or may be separate. The choke coils L_3 , if used, should be separate.

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Design formulas for ammeter.

MKS units.

This relation is used to convert inductance L to reactance $X = \omega L$:

$$\omega\mu_o = R_o(2\pi/\lambda) \qquad \text{ohms/meter} \qquad (2)$$

in which

ω = $2\pi f$ (radian/second)
μ_o = 1.257×10^{-6} henry/meter
R_o = 377 ohms
λ = wavelength (meters)

Air-core toroid with n circular turns ($n = 2$):

$$L_{12} = n\mu_o r_o{}^2/2r_2 \qquad (r_o^2 \ll r_2^2) \qquad \text{henrys} \qquad (3)$$

$$X_{12} = nR_o\ \pi r_o^2/r_2\lambda \qquad \text{ohms} \qquad (4)$$

in which

r_o = radius of circle of each turn (meters)
r_2 = radius of core circle through centers of turns (meters)

Total inductance of thermocouple heater circuit:

$$L_2 = L_2' + L_2'' = L_{12}(I_1/I_2) \qquad (5)$$

in which

L_2' = inductance of loop turns and leads (henrys)
L_2'' = inductance added in series with heater (henrys)
I_1 = current in center conductor (amperes)
I_2 = current in heater (amperes)

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Relative reduction of heater current I_2 by resistance in heater circuit:

$$\frac{1}{2}\left(\frac{R_2 + R_h}{X_2}\right)^2 \ll 1 \qquad (6)$$

Self-inductance of air-core toroid (for L_2''):

$$L = n^2\mu_o a^2/2b \qquad (a^2 \ll b^2) \qquad \text{henrys} \qquad (7)$$

in which

n = number of turns
a = RMS radius of circle turns (meters)
b = radius of core circle through centers of turns (meters)

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Ammeter component specifications.

The center conductor (I_1 , r_1) should be fairly straight in the neighborhood of the toroid, although the paired loops cancel out the first-order error that might be caused by departure from straightness.

The size of the center conductor may be reduced at the location of the toroid, enabling the use of a smaller toroidal shield.

The separation between center conductor and toroid should be several times the nominal breakdown distance in air (1 mm per 2 Kv RMS). Also a distance approximately equal to r_o is suggested as a minimum.

The toroidal shield should be made of a highly conductive material such as aluminum. At 14 Kc, the skin depth in aluminum is $\delta = 0.7$ mm . The wall thickness should be at least several times the skin depth. Clamping bolts may be made of any nonmagnetic material, and must be insulated at one or both ends to prevent any conductive path across the toroidal air space or the outer gap separated by insulation.

The mutual reactance should be sufficient to give a voltage, $V_2 = I_1 X_{12}$, which is much greater than the required heater voltage V_h , so the heater resistance R_h will have a negligible effect on the current I_2 through reactance X_2 . See formula (6). The current has a relative error from the mean, over the frequency range, which is less than $\pm$ 1/500 if $R_2/X_2 < 1/20$ and $R_h/X_2 = V_h/V_2 < 1/20$; this requires $X_{12} > 20\ V_h/I_1$.

In view of the preceding relations, it is advantageous to choose a thermocouple of moderately high current rating so that its required voltage is a small fraction of one volt. On the other hand, a higher current rating tends to require a higher volt-ampere rating for the series inductor L_2'' . The latter is a serious problem if an iron core is used, but less so if an air core.

The series inductance L_2'' must be nearly constant over the frequency range and temperature range. A rather small dielectric-core toroid, wound with Litz wire, is a preferred type. A somewhat larger coil of solid wire, of single-layer solenoid type in a shield can, is an alternative. An iron cup core, wound with Litz wire, may be helpful to reduce size, but requires careful design of airgap.

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Proposed design for ammeter.

Assuming that the inner conductor remains full size:

r_o	=	0.05 m = 2 in
r_1	=	0.125 m = 5 in
r_2	=	0.225 m = 9 in
r_2+2r_o	=	0.325 m = 13 in = outside radius over flange
n	=	2
f	=	14 Kc
λ	=	21.4 Km
L_{12}	=	.014 μh
X_{12}	=	1.23 milohm
I_1	=	4 Ka
I_2	=	4 a
I_2/I_1	=	1/1000
V_2	=	4.9 v
V_h	=	0.25 v
X_2	=	1.23 ohm
L_2	=	14 μh
I_2V_2	=	20 va (rating of L_2)
R_h	=	.062 ohm (1 watt)
R_2	=	.062 ohm (1 watt)

Design of toroid inductor for L_2'' :

$L = 14\ \mu h$; $R < 0.06$ ohm

a = .01 m = 0.4 in

b = .03 m = 1.2 in

n = 82 = 2 x 41 (2 halves of toroid)

$2\pi(b-a)/n$ = 1.53 mm = .060" = pitch on inside of toroid

$2\pi an$ = 5.2 m = length of wire

Litz wire equivalent to 14 gauge, about 1 layer on outside and 2 layers on inside of toroid.

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Ammeter components and details.

Thermoelement:

Weston Model 9997, Type C, 4 amp., round case, $17.75.

Delivers 12-15 mv. output at rated current input.

Electronic Milli-Volt/Ammeter (DC):

Weston Model 1477, in stock, $425.

Use scale 10 mv for 4000 amp.

5 " " 2800 "

Calibrate on standard scale (100 div.).

Later, order special scales (direct reading).

Thermo Ammeter (for calibration purposes):

Weston Model 622.2901004, 50 amp., $309.

Use with 80 turns for 4000 amp.

56 " " 2800 ".

Current ratio: The choice of a thermoelement was governed by several factors. (1) The higher-current ranges are more rugged, avoiding vacuum seal and having a lower-resistance thermocouple that is less sensitive to burnout. (2) These ranges come in Type C, which is compensated against ambient temperature variations. (3) The higher ranges of Type C are dependent on thermal conduction in the heater and independent of air cooling, which removes the effect of air temperature. The ranges from 3.5 amp upward have mainly conductor cooling, as indicated by the lowest voltage (0.2 v) at rated maximum current. The lower ranges have higher voltage, up to 0.4 v for the 0.5 amp range, the latter indicating air cooling about equal to conductor cooling. The range of 4 amp was chosen as about the lowest that would rely mainly on conductor cooling.

Current ranges: The thermoelement is rated to deliver:

12-15 mv at 4 amp.

10 ± 1.1 " " 3.44 "

Therefore it is proposed to operate at the following full-scale levels:

10 mv	3.5 amp	(4000 amp)
5 "	2.5 "	(2800 ")

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This matches two of the standard scales on the Model 1477. This requires a current ratio slightly different from that assumed on p. 12.

Series inductor: The idea is to ehoose a series inductor that will give 4000 amp full-scale for 10 mv from the thermoelement. The best estimate yields these changes from p. 12:

I_2 = 3.5 amp (for 10 $\pm$ 1 mv from thermoelement)
I_2/I_1 = 1/1140
V_h = $R_h I_2$ = 0.18 v
X_2 = 1.40 ohm
L_2 = 16 μh
$I_2 V_2$ = 17 va (rating of L_2)
R_h = .05 ohm (0.6 watt)
R_2 = .09 ohm (1 watt) or less

Design of toroid inductor for L_2'' (changes only):

L = 16 μh ; R < .09 ohm
n = 88 = 2 x 44 (2 halves of toroid)
$2\pi an$ = 5.5 m = length of wire

Pair of loops: In Fig. 2, note cross-over on one side in connection of two loops in series.

Grounding and shielding: Omit all ground connections for heater AC or junction DC circuits. Omit series choke coils. At each end of long line, ground each terminal through 0.1 μf in series with 50 ohms; the principal purpose is to damp the long line against resonance at harmonic frequencies. In each output lead from thermoelement, insert a fuse for 0.1 amp; it is estimated that 0.5 amp is about the maximum safe current in junction leads, but the actual current is mainly the meter current and the charging current in bypass capacitors and long line (about 0.1 μf across the circuit); these currents are much less than the suggested fuse rating.

Long line: Pair of insulated wires in a metal pipe (any metal). Total resistance less than 50 ohms, for best operation of remote meter.

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Remote meter: Model 1477 includes 200 Kc amplifier operated on AC power. There is a question of susceptibility to 200 Kc harmonics of transmitter. This will be investigated and may require special shield box and 200 Kc filtering, in view of high power nearby. Linearity and stability are within 0.1 % of full scale for input circuit resistance within 50 ohms. This applies to input current squared. Proposed scales (above) are based on normal operation and calibration near mid-scale.

Space: The following are the approximate overall dimensions of components.

Thermoelement, round case, 3" dia. x 2" ht.

Meter-amplifier, 8" width x 7" ht. x 5" depth.

Note added later.

Tests on remote meter: Model 1477 has been borrowed from Weston and tested for susceptibility to AC voltage superimposed on DC voltage being measured. The following conclusions were reached, relating to the 10 mv and 5 mv scales that are intended to be used.

(1) At frequencies around 200 Kc, .01 v AC may cause a transient kick of ± 1 or 2 divisions (on 100-division scale) but no residual error as great as 0.1 divisions; 0.1 v causes a kick of ± 12 or 25 div and residual error less than 0.5 div.

(2) At frequencies of 15-30 Kc, 0.1 v AC causes no error as great as 0.1 division.

It is concluded that superimposed AC voltage is tolerable up to 0.1 v at 15-30 Kc and up to .01 v near 200 Kc (of interest when this is a harmonic of the operating frequency).

NB 100W, p. 143.

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Variometer inductance computations.

Symbols (MKS units):

a	=	radius (meters)
b	=	n x pitch of winding, axial length (meters)
n	=	number of turns
L	=	inductance (henries)
μ_o	=	1.257×10^{-6} = magnetivity of free space (henries/meter)
sub-1	=	inner coil
sub-2	=	outer coil
sub-12	=	mutual

Inductance of helix (solenoid):

$$L = \frac{\pi \mu_o a^2 n^2}{b + 0.9\ a} \tag{8}$$

Mutual inductance of two concentric coaxial helices, if inner coil is slightly smaller than outer coil:

$$L_{12} = L_2 \frac{a_1^2\ n_1}{a_2^2\ n_2} \tag{9}$$

Coefficient of coupling of two concentric spherical coils:

$$k_{12} = (a_1/a_2)^{3/2} \tag{10}$$

In the present case, the coils are incomplete spherical coils with a few turns of varying pitch, so the helix approximation requires some judgment in choice of effective radius and length.

n_1	=	4	n_2	=	5
a_1	=	1.66 m	a_2	=	1.75 m
b_1	=	1.90 m	b_2	=	2.80 m
L_1	=	51 ± 2 μh	L_2	=	70 ± 3 μh
L_{12}	=	49 ± 2 μh	$4L_{12}$	=	196 ± 8 μh

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With the two coils connected in series, the last value is the variation of inductance to be obtained by rotation of 1/2 turn. The self-inductance is augmented by the leads, ignored in these formulas, which may be regarded as decreasing the coefficient of coupling.

The coefficient of coupling happens to come out the same by formulas (9) and (10):

$$k_{12} = 0.62$$

H. A. Wheeler, "Simple inductance formulas for radio coils", Proc. IRE, vol. 16, p. 1398-1400; Oct. 1928.

H. A. Wheeler, "The spherical coil as an inductor, shield or antenna", Proc. IRE, vol. 46, p. 1595-1602; Sept. 1958.

CEMCO Dwg. No. 28092-R.

NB 100W, p. 125-8.

ew

Transmission-line power and reflection meter.

A directional coupler is proposed, which offers some attractive features. It has a minimum of electronic components (only semiconductor diodes) and requires no power supply. The direct and reflected power and voltage calibrations are independent of frequency. Fig. 4 shows the circuit.

The voltage and current couplings from the main line are provided by C_{12} and L_{12} . The forward power couples into one network and the backward (reflected) power into another.

Since the couplings are proportional to frequency, it is necessary to provide in each network an opposite variation with frequency, as shown in Fig. 5. The entire network is built out to give constant R at I_2 , V_2 . This provides a voltage V_3 which is compensated against frequency variation. Also it incidentally provides another voltage V_4 in phase quadrature with V_3 .

The compensated voltage V_3 is applied to AC voltmeter M_3 to indicate forward power and corresponding voltage. The other compensated voltage V_3' is applied to voltmeter M_3' to indicate reflected power and corresponding voltage. The voltage ratio of the latter over the former is the reflection coefficient of the load.

The in-phase compensated voltage V_3 and the quadrature-phase voltage V_4 are used as phase references for deriving the respective components of the reflected voltage. The latter are indicated on zero-center DC meters M_{33} and M_{34} . Each of these meters has two ranges. The upper range gives full scale at the maximum voltage of M_3 and M_3' ; the lower range at half the voltage. Since the scales are all linear, no greater expansion may be needed.

The table herewith gives typical values for all components.

The main coupler (C_{12} , L_{12}) is not completely designed, since there is a problem coupling sufficient power in a small space (8-32 w at 15-30 Kc). The arrangement is feasible with some refinement of the coupler design.

The frequency compensation can be obtained by adjusting C_3 to give equal V_3/V_1 at the lowest and highest frequency. Then L_4 in the other branch is adjusted to give constant R of the two branches in parallel. This compensation should be good within ±0.5 percent of the power reading.

The in-phase and quadrature components are selected by discriminators and linear rectifiers in a manner that is independent of the amplitude of the phase reference voltages (marked 0° and 90°), provided these voltages are sufficiently great, which is assured in the circuit shown.

NB 100W, p. 144-150. NB 104W, p. 3-9.

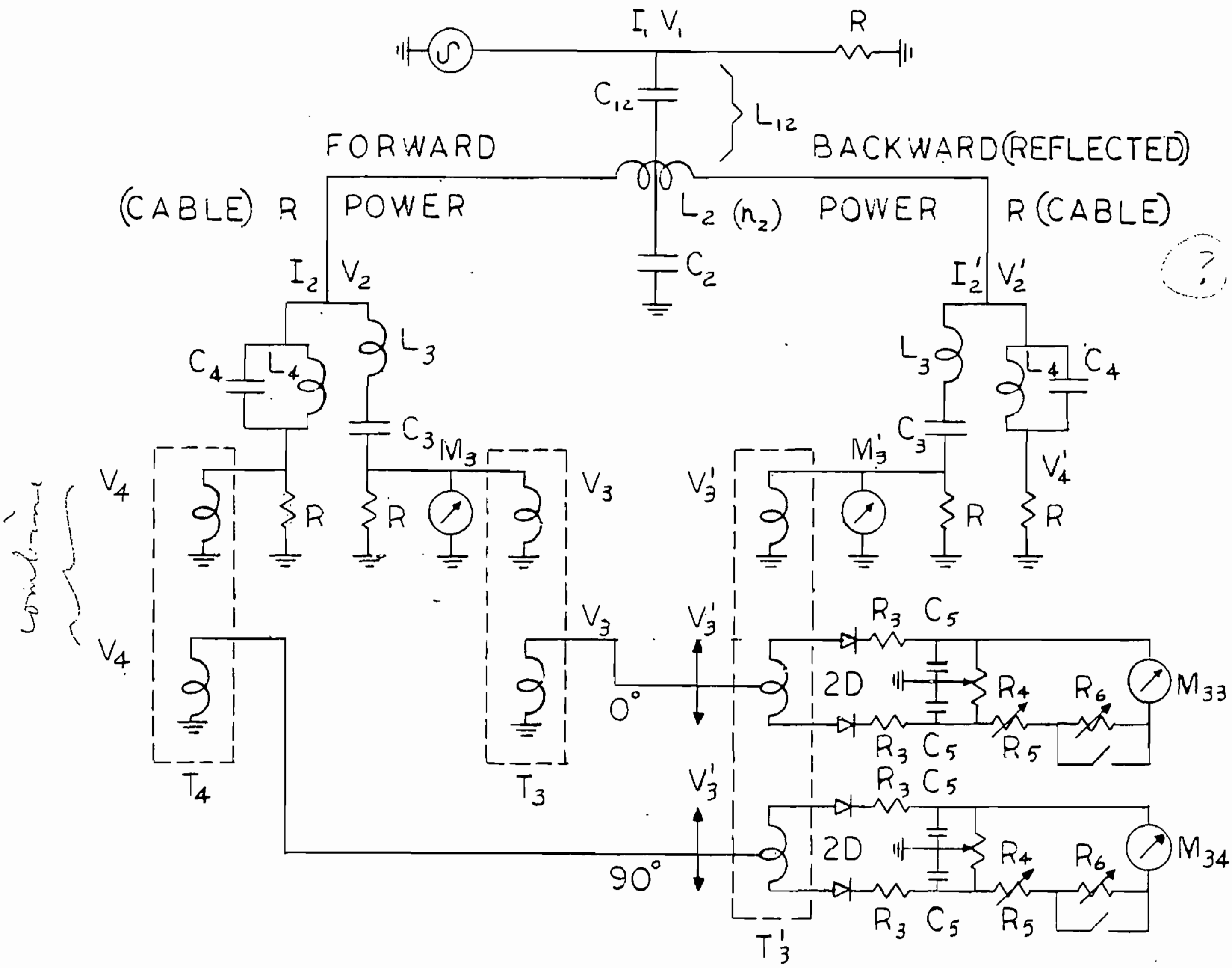
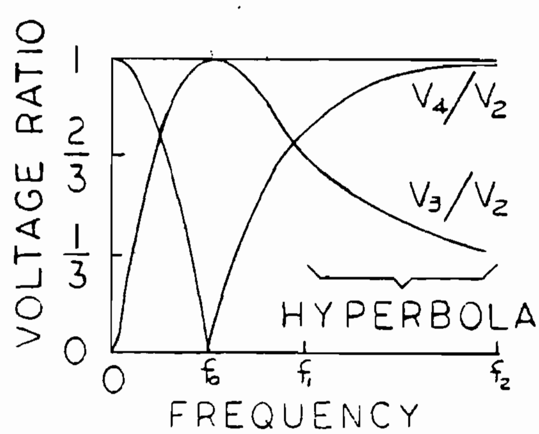


Fig. 4 - Circuit diagram of power and reflection meter.



ew Fig. 5 - Voltage ratio inversely proportional to frequency.

R = 50 ohms (26 watts max in each of 4 units)
C_{12} = 382 μμf
C_2 = $(n_2 - 1)C_{12}$ = .0378 μf
L_{12} = 0.95 μh
L_2 = $n_2 L_{12}$ = 95 μh (air core toroid around I_1 conductor)
n_2 = 100 turns on L_2

L_3 = 0.80 mh
C_3 = 0.65 μh (approx.)
C_4 = .035 μh
L_4 = 15 mh (approx.)
L_3C_3 = L_4C_4 = resonant at f_o
f_o = 7 Kc (approx.)
f_1; f_2 = 15; 30 Kc = nominal limits of operating frequency range
R_3 = 5 K-ohms
R_4 = 2 K-ohms (adjustable center tap)
R_5 = 4 K-ohms (adjustable, scale multiplier)
R_6 = 2 K-ohms (adjustable, scale calibrator)
C_5 = 0.1 μf (RF bypass)

D = semiconductor diode, half-wave rectifier, peak current 2 ma, peak inverse voltage 100 v; may use two in series at each location.

T_3 , T_3' , T_4 = transformers, 2 or 3 equal windings, some center-tapped; self-resonant or tuned near 20 Kc by shunt C on primary (across R).

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Current and voltage RMS values based on max power:

I_1 = 200 amp
V_1 = 10 Kv
I_1V_1 = 2000 Kw
V_2 = V_2' = 18-36 v at 15-30 Kc
I_2 = I_2' = 0.36 - 0.72 amp at 15-30 Kc
V_3 = V_3' = 12 v (independent of frequency)
V_4 = V_4' = 13-34 v at 15-30 Kc, in phase quadrature relative to V_3

M_3 = Weston 1332 AC microammeter (rectifier type) 500 μa, 750 ohms, with series resistor about 23 K-ohms for 12 v RMS full-scale (3.25" scale); prefer larger meter with similar specifications, such as Model 1977 (special, 7.2" scale). Scales for power (2000 Kw, unusual scale) and voltage (10 Kw, nearly linear scale) on 50-ohm load; read "forward" power.

M_3' = Weston 1332, see M_3 ; no need for larger meter unless desired to match. Similar scales; read "backward" or "reflected" power.

M_{33} = Weston 1331 DC microammeter 500 μa, 230 ohms, zero center. Scales for 10-0-10 Kv and 5-0-5 Kv on 50-ohm load; read "reflected" voltage, in-phase component.

M_{34} = same as M_{33} ; read "reflected" voltage, quadrature component.

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Directional coupler design.

The directional coupler included in Fig. 4 can be improved by modification shown in Fig. 6. The latter may be constructed as shown in Fig. 7. The result is a design that offers essential simplicity and constant calibration over the frequency range. (For convenience in computation, the 1:2 frequency range of 15-30 Kc is assumed, but the same design is good for 14-30 Kc.)

Referring to Fig. 6, the voltage coupling is obtained by a lumped capacitance C_{12} and the current coupling by mutual inductance L_{12}. The secondary circuit C_2L_2 simulates a distributed line by 2 sections of a low-pass filter having a cutoff frequency f_c much higher than the operating frequencies. Both couplings are introduced effectively at the same point in the secondary line, since the current coupling is independent of the middle shunt capacitor $\frac{1}{2}C_2$. Therefore, for a pure traveling wave, the two couplings are in aiding phase toward one side (the left side, V_2) and are in opposing phase toward the other side (the right side, V_2'). Making the two couplings equal at one frequency, they remain equal to the extent that the secondary circuit approximates a smooth line. These relations provide the action of a directional coupler.

The lumped structure of the secondary line causes a departure from constant R, whose first approximation has the magnitude

$$|\Delta Z/R| = \frac{1}{32}(\omega L_2/R)^3 \tag{11}$$

This is an upper limit on the error in reflection coefficient that can be caused by the lumped line. It happens that this error can be reduced by trimming $\frac{1}{2}C_2$, leaving a residual error within the following limit:

$$\frac{1}{128}(\omega_2 L_2/R)^3 \tag{12}$$

Complete directivity at one frequency can be obtained by trimming both $\frac{1}{2}C_2$ and either C_{12} or L_{12}, to give zero indication of reflection (V_3') from the nominal load (pure $R = 50$ ohms).

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Fig. 7 shows a shielded structure for realizing this directional coupler.

The voltage-coupling capacitor C_{12} may be made of a metal sleeve over an insulating sleeve over the main conductor, as shown. (For electric properties, the metal sleeve may be continuous around its circumference.) Alternatively, this capacitor may be a mica unit mounted on the center conductor.

It operates normally at 10 Kv RMS, and may be subject to higher values during adjustments or abnormal conditions, so a peak rating of 40 Kv RMS or a working rating of 20 Kv RMS is suggested, for the operating frequency range.

The current-coupling inductance L_{12} is obtained by mutual inductance to a pair of toroidal coils (each $\frac{1}{2}L_2$) surrounding the main conductor. They are wound of Litz wire on cores of insulating material (non-magnetic). They are enclosed in a shield box of toroidal shape, made of sheet metal (non-magnetic). This box has an opening on the inside, to avoid any effect on the inductive coupling. This shield box is grounded through the outer conductors of the pair of cables (R) which carry power from the directional coupler to the meter-circuit box (the remainder of Fig. 4).

As mentioned previously, C_3 (and consequently L_4) is adjusted to give the same calibration at both limits of the frequency band, thereby assuring nearly constant calibration over the band.

The following formulas include the principal relations to be used in designing this directional coupler and associated RF circuits. (See table for symbols.)

$$R = V_1/I_1 = V_2/I_2 \tag{13}$$

$$R^2 = L_{12}/C_{12} = L_2/C_2 = L_3/C_4 = L_4/C_3 \tag{14}$$

$$\omega L_{12} = V_2/I_1 \; ; \; \omega C_{12} = I_2/V_1 \tag{15}$$

$$f_1/f_2 = 1/2 \; ; \; f_o/f_2 = 1/\sqrt{18} = 0.236 \tag{16}$$

$$\omega_2 L_3 = 3R \tag{17}$$

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At f_1 to f_2 ; $V_3/V_2 = 2/3$ to $1/3$ (18)

$$V_3/V_1 = L_{12}/L_3 = C_{12}/C_4 \text{ (constant)} \quad (19)$$

$$C_3/L_3 = C_4L_4 = 1/\omega_o^2 \quad (20)$$

$$C_2L_2 = 16/\omega_c^2 \quad (21)$$

The following formulas relate to the toroidal coils (non-magnetic core, single layer winding):

$$\tfrac{1}{2}L_2 = \tfrac{1}{2}L_{12}(\tfrac{1}{2}n_2) = \mu_o \frac{a^2}{2b}(\tfrac{1}{2}n_2)^2 \quad (22)$$

$$\tfrac{1}{2}n_2 = \frac{L_{12}b}{\mu_o a^2} = L_2/L_{12} = C_2/C_{12} \quad (23)$$

$$\text{Inside pitch of winding} = \frac{2\pi(b-a)}{\tfrac{1}{2}n_2} \quad (24)$$

a = radius of each turn of toroidal coil
b = radius of core centerline circle of toroidal coil
$\tfrac{1}{2}n_2$ = number of turns on each coil ($\tfrac{1}{2}L_2$)
μ_o = 1.257 μh/m = magnetivity of free space

The following values are proposed as the basis for a design:

V_3/V_1 = .001 = calibration ratio (constant)
V_1 = 10 Kv RMS
I_1 = 200 amp RMS
V_2 = 15-30 v at 15-30 Kc
I_2 = 0.3 - 0.6 amp
I_2V_2 = 4.5 - 18 watts
V_3 = 10 v (constant)
V_4 = 11 - 28 v at 15-30 Kc

f_1 = 15 Kc = lower limit of operating range
f_2 = 30 Kc = upper limit of operating range
f_o = 7 Kc = resonant frequency below operating range
f_c = 520 Kc = cutoff frequency of lumped line (C_2L_2)
ω = $2\pi f$ (same subscripts)

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C_{12} = 320 μμf = voltage-coupling capacitance
C_2 = .0244 μf
C_3 = 0.64 μf
C_4 = 0.32 μf

L_{12} = 0.80 μh = current-coupling mutual inductance
L_2 = 61 μh
L_3 = 800 μh
L_4 = 1600 μh
n_2 = 152
a = 25 mm
b = 75 mm

R_3 = 4 K-ohms

See pages 21-22 for the following:
R_4 , R_5 , R_6 , C_5 , D , T (all), M (all);
M_3 (etc.) change to 10 v RMS full-scale.

NB 104W, p. 10-16.

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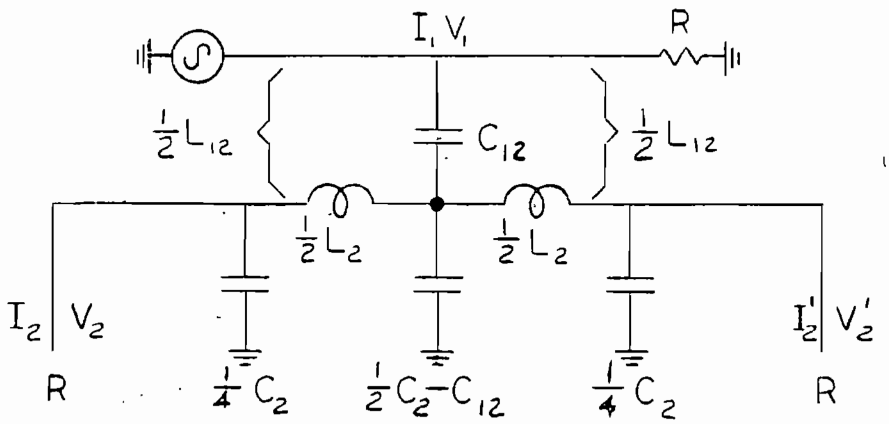
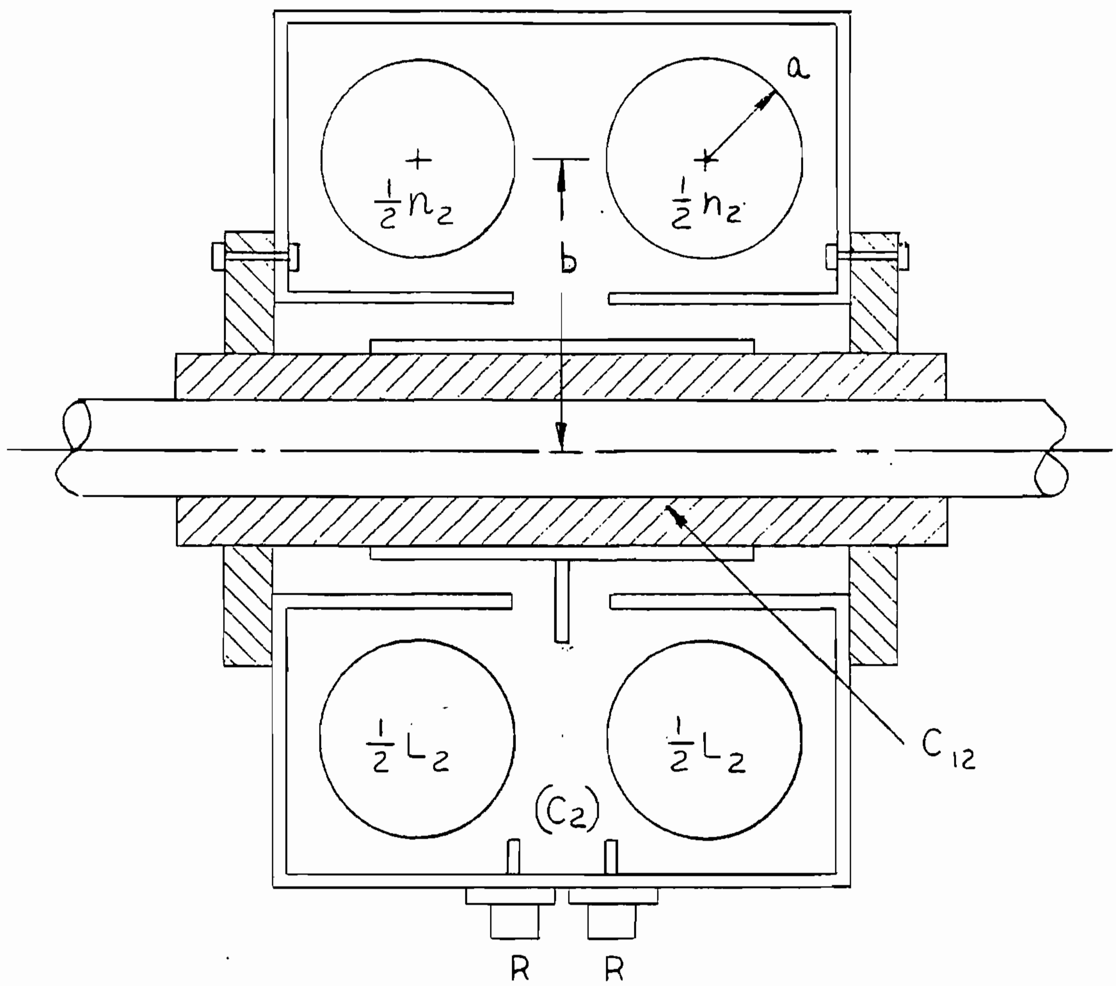


Fig. 6 - Circuit details of directional coupler.



ew Fig. 7 - Structure details of directional coupler.

Directional coupler for 100-ohm line.

The preceding formulas and computations, referring to Fig. 6, will be revised to accommodate different values for the main-line wave resistance here denoted, R_1 (at the top), and the auxiliary-cables wave resistance, here denoted R_2 (at the bottom). The formulas that are revised are designated by their original numbers followed by "a".

$$|\Delta Z/R| = \frac{1}{32}(\omega L_2/R_2)^3 \tag{11a}$$

$$\frac{1}{128}(\omega_2 L_2/R_2)^3 \tag{12a}$$

$$R_1 = V_1/I_1 \; ; \; R_2 = V_2/I_2 \tag{13a}$$

$$R_1 R_2 = L_{12}/C_{12} \; ; \; R_2^2 = L_2/C_2 = L_3/C_4 = L_4/C_3 \tag{14a}$$

$$\omega_2 L_3 = 3\, R_2 \tag{17a}$$

$$V_3/V_1 = (R_2/R_1) L_{12}/L_3 = C_{12}/C_4 \quad \text{(constant)} \tag{19a}$$

The following values are proposed as the basis for a design:

V_1 = 15 Kv RMS
I_1 = 150 amp RMS
$V_1 I_1$ = 2250 Kw
V_2 = 15-30 v at 15-30 Kc
I_2 = 0.3-0.6 amp
$V_2 I_2$ = 4.5-18 watts
V_3/V_1 = 1/1500 = calibration ratio (constant)
V_3 = 10 v (independent of frequency)
V_4 = 11-28 v

f_1 = 15 Kc = lower limit of operating range
f_2 = 30 Kc = upper limit of operating range
f_o = 7 Kc = resonant frequency below operating range
f_c = 520 Kc = cutoff frequency of lumped line ($C_2 L_2$)

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R_1	=	100 ohms	L_{12}	=	1.06 μh
R_2	=	50 ohms	L_2	=	107 μh
			L_3	=	800 μh
C_{12}	=	212 μμf	L_4	=	1600 μh
C_2	=	.043 μf	n_2	=	202
C_3	=	0.64 μf	a	=	25 mm
C_4	=	0.32 μf	b	=	75 mm
R_3	=	4 K-ohms			

See pages 21-22 for the following:
R_4 , R_5 , R_6 , C_5 , D , T (all), M (all);
M_3 (etc.) change to 10 v RMS full-scale.

NB 104W, p. 17-19.

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Calibration of power meter.

For calibration of the power meter, an imperfect load impedance may be used by the following procedure.

Make the load impedance $100 \pm 10 \pm j10$ ohms. The reflection coefficient magnitude is within .07, so the reflected power is within .005 of the direct power.

Observe the voltage and current at the load. Their product is within .005 of the load power (the direct power minus the reflected power).

If these errors are ignored, the direct-power meter can be calibrated directly in terms of load volt-amperes, with a relative error within .01 of the power, or .005 of the voltage. By holding the load within 1/2 the specified tolerances, the error can be held within 1/4 of these values.

The reflection meters should be balanced to zero on a load impedance close to the nominal value of 100 ohms pure resistance. If such a load is not available for high power, the same result may be approximated as follows, which is probably close enough for practical purposes.

To balance the in-phase component, observe this reflection meter M_{33} with the load on open-circuit (15 Kv) and then on short-circuit (150 amp). Adjust L_{12} (or C_{12}) to equalize the two readings at opposite sides of the zero-center. Adjust $R_5 + R_6$ to give full scale on both sides. (The difference $R_5 - R_6$ is adjusted with reduced reflection to give the scale reading x 2 when R_6 is switched out. The center-tap on R_4 is adjusted for zero-center while the primary of T_3' is disconnected from M_3'.)

To balance the quadrature component, observe this reflection meter M_{34} with the load on OC or SC, as above. Trim $\frac{1}{2}C_2$ to give zero-center on the meter, which should occur on both OC and SC alike.

The M_{33} and M_{34} circuits are nominally alike, except for the 0° and 90° phase references. Therefore these references may be interchanged for making zero-center or scale-calibration adjustments on either meter. For example, the in-phase procedure may be used for both meters by making this interchange.

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Graphical presentation of antenna reactance.

Assuming that an antenna behaves as lumped L and C in series, its reactance is

$$X = \omega L - 1/\omega C \; ; \quad \omega_o^2 CL = 1 \tag{25}$$

$$-X/\omega = L\,(\omega_o^2/\omega^2 - 1) \tag{26}$$

This latter form becomes a straight line if graphed as in Fig. 8. Any two points on the line are sufficient to determine C and L .

The best two points are at ω_o and ω_1 , the former being merely a test of resonance and the latter a test of nearly pure C . For more than two points at frequencies below resonance, all points should fall on a straight line.

NB 104W, p. 20.

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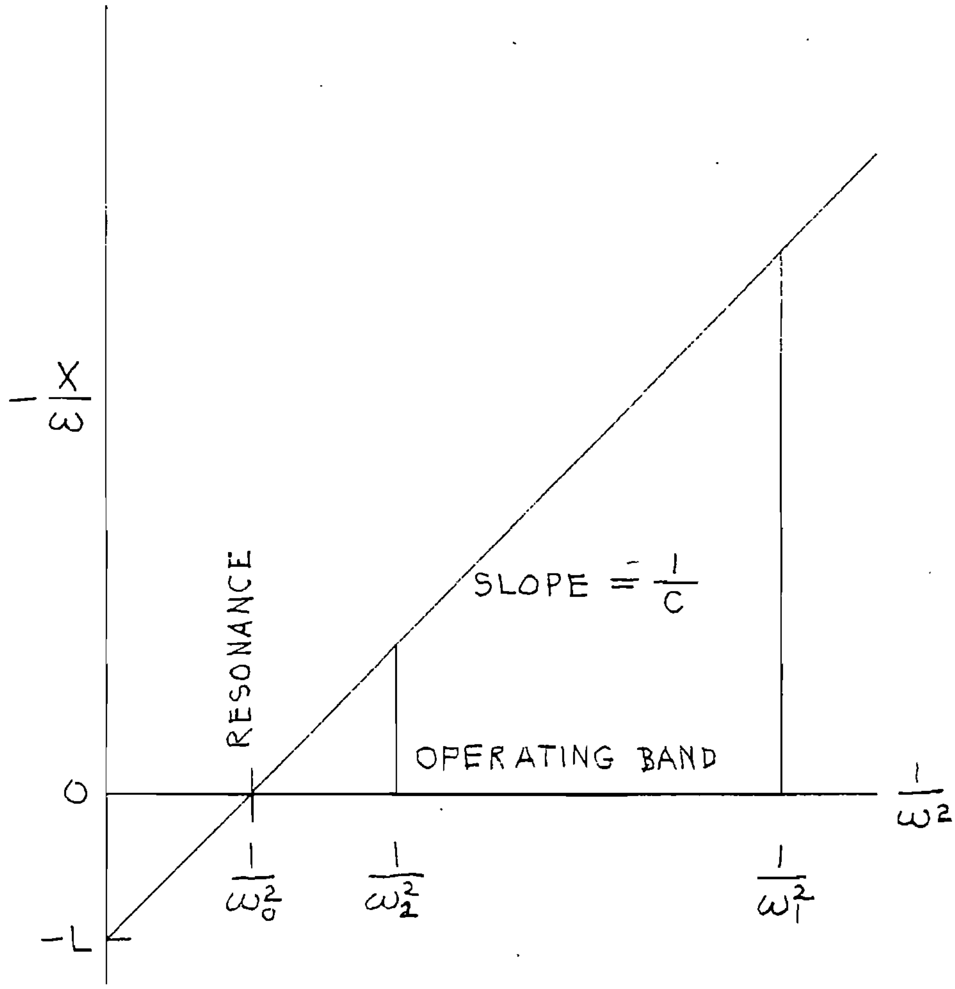


Fig. 8 - Straight-line graph of antenna reactance.

Report 1523

VLF TRANSMITTER NOTES

By Harold A. Wheeler and Richard F. Frazita

To Continental Electronics Systems, Inc.

1967 DEC 28 Job 540

NB 128W, pp. 77-110; NB 2093, pp. 75-113.

mc

Contents

This is a cumulative report for collecting information developed in the course of studies and consultation relating to the VLF transmitter in Italy for NATO. The following list will be brought up to date and reissued with each release of additional pages.

mc

Ground system comprising grid of parallel wires above the surface.

The subject antenna is suspended from three mountain peaks. The ground surface underneath ranges from sea water to rocky terrain. The latter, even if provided with radial buried wires, causes substantial loss associated with the vertical E field. It is proposed to reduce such losses by supporting the radial wires somewhat above the surface.

A theory has been developed, based on a planar grid of parallel wires suspended above a planar surface of homogeneous earth. For any dielectric properties of the earth, there is some value of conductivity which causes maximum loss. This is termed the "worst" conductivity. It is found possible to reduce to a very small value the loss caused in such a case, so that even this extreme does not impose an excessive burden on the cost.

In examples to be given, the following simple rules are found to hold the E-field losses down to a tolerable or negligible value.

(1) Space the radial wires just close enough to hold down the H-field losses.

(2) Choose the wire size and height of suspension for economy of installation and maintenance, the height being sufficient to clear the surface by some margin (such as 1 meter).

(3) Note that non-metallic posts or poles are preferable for suspension.

(4) Note that the average height of the wires is subtracted from the effective height of the antenna.

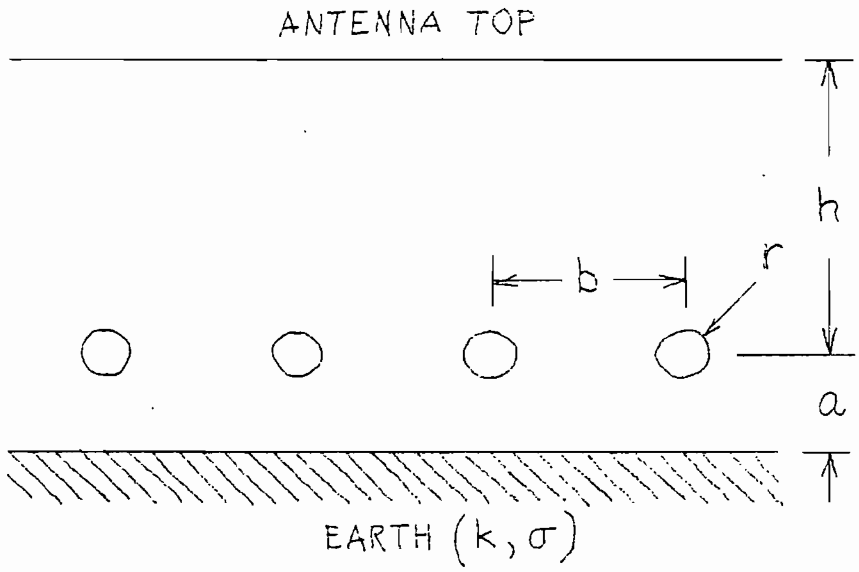
(5) Note that increasing the wire spacing (and, in a lesser degree, decreasing the wire size) has the effect of decreasing the antenna capacitance and the frequency bandwidth.

There will be given a basic formula for a simple model, then a procedure for practical application.

H.A.Wheeler, "VLF antenna notebook - ground system", WL Report 304 to DECO, Oct. 1956. (The original study of E-field area losses during design of antenna at Cutler, Me.).

mc

Theoretical formula for E-field losses.



Conditions:

Parallel-plane antenna capacitor extending indefinitely in horizontal dimensions, so that edge effects can be ignored.

Plane grid of parallel wires, as lower plate of capacitor.

Wire radius small relative to spacing from surface and each other ($r/a \ll 1$; $r/b \ll 1/4\pi$).

Wire separation small relative to antenna height ($b/h \ll 1$).

Wires of perfect conductivity.

Homogeneous isotropic earth (k, σ).

"Worst" conductivity in the earth: $\sigma = \omega\varepsilon_o(k+1)$ approximately.

p = power factor of antenna capacitance, caused by E-field losses in ground system.

a, b, h, r = space dimensions (meters).

σ = conductivity in earth (mhos/meter).

k = electric ratio in earth, relative to free space.

$$p = \frac{1}{k+1} \frac{\exp -4\pi a/b}{4\pi h/b + 2 \ln b/2\pi r} < \frac{1}{k+1} \frac{b}{4\pi h} \qquad (1ab)$$

From the simple form, p is further decreased by increasing a or decreasing r, either of which decreases the radiation bandwidth.

This formula is independent of frequency, except for the incidental frequency dependence of the effective values of the earth properties and their relation to "worst" conductivity.

As applied to the entire antenna, there are several other factors, each of which substantially decreases p from its value in the idealized model, as follows:

(1) The spacing of the top wires.
(2) The edge effects in the area under the antenna.
(3) The fraction of area covered by sea water, a more favorable ground conductor.

The formula can be applied, with reduction factors, to any part of the area of influence under the actual antenna.

mc

Practical formula for E-field losses.

MKS units. See p. 4.

h_a = average height of top wires above ground wires.

c' = apparent increment of height related to spacing and radius of top wires, as decreasing the E field at the ground.

c = apparent increment of height related to spacing and radius of ground wires.

h = $h_a + c'$

A' = area covered by grid of top wires.

l' = total length of top wires.

b' = A'/l' = average spacing of top wires.

r' = average radius of top wires.

k_a = fraction of total area (under top wires) covered by ground wires.

k_p = reduction factor applied to E losses as a result of divergent E field on ground under top area.

p_a = power factor of antenna, contributed by E losses in area covered by ground wires.

$$p_a < \frac{k_a k_p}{k+1}\,\frac{b}{4\pi(h_a + c' + c)} < \frac{k_a k_p}{k+1}\,\frac{b}{4\pi h_a} \qquad (2ab)$$

The height increments have a minor effect, but may be computed as follows:

$$c = \frac{b}{4\pi}\ln b/4\pi r \quad ; \quad c' = \frac{b'}{4\pi}\ln b'/4\pi r' \qquad (3ab)$$

NB 128W, pp.77-110.

mc

Example of E-field losses:

	h_a	=	180 m
	a	>	1 m
	b	=	10 m
	r	=	2 mm
	k	=	9
	k_a	=	1/2
	k_p	=	1/2
(2b)	p_a	<	.00011 = 0.11 mil
"worst"	σ	=	10 μmho/m at 18 Khz (lowest frequency)

This example is illustrative of the subject antenna. The computed value of the upper bound of p_a may be compared with the radiation power factor of entire antenna without downleads, which is about 0.7 mil at the lowest frequency. It is seen that the assumed conditions give a negligible amount of E-field losses. Therefore closer computation is not needed.

The "worst" conductivity is seen to be much less than typical values to be expected, so this ratio further decreases the E-field losses in the area of ground covered by the wire grid.

mc

Small Antennas

HAROLD A. WHEELER, LIFE FELLOW, IEEE

Abstract—**A small antenna is one whose size is a small fraction of the wavelength. It is a capacitor or inductor, and it is tuned to resonance by a reactor of opposite kind. Its bandwidth of impedance matching is subject to a fundamental limitation measured by its "radiation power factor" which is proportional to its "effective volume". These principles are reviewed in the light of a quarter-century of experience. They are related to various practical configurations, including flush radiators for mounting on aircraft. Among the examples, one extreme is a small one-turn loop of wide strip, tuned by an integral capacitor. The opposite extreme is the largest antenna in the world, which is a "small antenna" in terms of its operating wavelength. In each of these extremes, the radiation power factor is much less than one percent.**

I. Introduction

A "SMALL ANTENNA" is here defined as one occupying a small fraction of one radiansphere in space. Typically its greatest dimension is less than $\frac{1}{4}$ wavelength (including any image in a ground plane). Some of its properties and available performance are limited by its size and the laws of nature. An appreciation of these limitations has proved helpful in arriving at practical designs.

The radiansphere is the spherical volume having a radius of $1/2\pi$ wavelength [10]. It is a logical reference here because, around a small antenna, it is the space occupied mainly by the stored energy of its electric or magnetic field.

Some limitations are peculiar to a passive network, where the concepts of efficiency, impedance matching and frequency bandwidth are essential and may be the controlling factors in performance evaluation. This discussion is directed mainly to these limitations in relation to small size. This subject has been on the record for a quarter-century but is still too little taught and appreciated. It centers around the term, "radiation power factor" and its proportionality to volume [2].

As in any area of engineering compromise, there have been some ingenious developments for realizing some qualities at the expense of others. A valid comparison of alternatives requires careful description and evaluation in terms of well defined quantities, especially in the use of terms such as efficiency and impedance matching. Also in the size comparison of circuits qualified for high power or low power [11].

An outline of some of the relevant principles will be followed by a brief reference to the background in the use of an amplifier with a small antenna for reception. Then the principal topic will be introduced in terms of the bandwidth limitations of impedance matching with a resonant circuit, which is a tuned antenna circuit in this discussion.

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TABLE I
COMPARISON OF TOPICS OF EFFICIENCY AND AMPLIFICATION

EFFICIENCY	TOPIC	AMPLIFICATION
PASSIVE	LINEAR NETWORK	ACTIVE
ESSENTIAL	IMPEDANCE MATCHING	OPTIONAL
NO	TOLERANCE OF LOSSES	YES
THERMAL	NOISE	AMPLIFIED
NO	POWER LIMITING	YES

The radiation power factor will be reviewed in concept and in some applications to typical antennas in the form of capacitors and inductors. Some special applications will be described for flush mounting and for VLF transmission and reception. In every case, the efficiency and/or bandwidth is seen to be limited ultimately by size.

II. Principles

Table I shows a comparison between efficiency and amplification, referring to some topics relevant to small antennas. Its purpose is to emphasize the distinction between efficiency and amplification, the former being the basis for this presentation. The relations in this table may help to bring out the accepted meanings of various terms.

Efficiency implies the utilization of the amount of radiated signal power that can be intercepted by the receiver. If the antenna is small, the greatest power transfer to a circuit requires impedance matching. This is achieved in a passive network by tuning the antenna and coupling to the circuit.

Amplification implies the utilization of the intercepted signal, but the excitation of the amplifier may not require impedance matching in the active network. This may facilitate a wideband design, as in one example to be shown. However, the amplifier may add much to the thermal noise generated in the antenna dissipation.

In a linear network, efficiency is associated with a passive network, while amplification is associated with an active network. In a weak-signal receiver, linearity is not a primary problem. In a power transmitter, however, an active network imposes an upper limit.

In general, efficiency is reduced by losses. This is particularly true in a small antenna where the radiation power factor is small and may be far exceeded by the loss power factor. In a weak-signal receiver, an amplifier can make up for losses in respect to signal strength, but only with increasing background of thermal noise. In a power transmitter, the power rating must be increased to cover losses.

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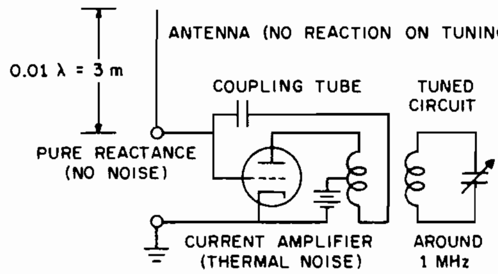

Fig. 1. Small antenna with wideband coupling tube, used in broadcast receivers (1928).

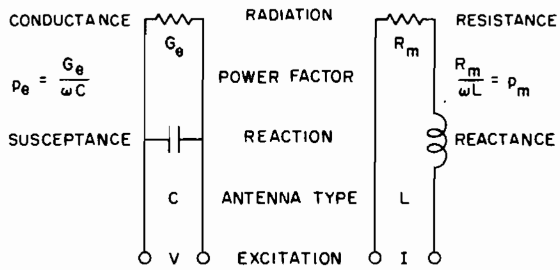

Fig. 2. Radiation power factor of small antenna.

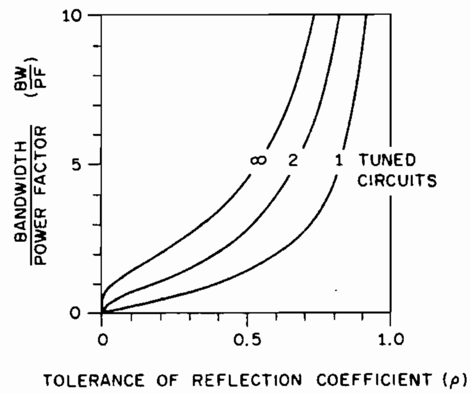

Fig. 3. Bandwidth of matching with tuned circuits.

These relations are emphasized because there have been some invalid ratings of small antennas associated with active devices serving as amplifiers. The greatest confusion has been associated with transmitters, by ignoring the power limitations imposed by small active devices. These limitations are not avoided by any particular relation between the small antenna and the amplifier.

III. Background

The wideband utilization of a small antenna was accomplished in a receiver about a half-century ago. That history is relevant to the more recent proposals using an amplifier in conjunction with a small antenna [11].

Fig. 1 shows a circuit that was commonly used in radio broadcast receivers about 1928. It operated over a frequency ratio of 1:3. A short wire is simply connected to the grid of the first tube. It bears a striking resemblance to some recent proposals, but using a tube instead of a transistor, and at lower frequencies. It substituted amplification for antenna tuning. It increased the noise threshold and also suffered from crossmodulation of all signals by any one strong signal. Then the pendulum swung and it was superseded by double tuning ahead of the first tube. The tuning yielded efficiency over noise and also preselection against crossmodulation.

IV. Frequency Bandwidth of Impedance Matching

There are limitations on the frequency bandwidth of impedance matching between a resonant circuit (antenna) and a generator or load. A quarter-century has elapsed since these limitations were developed and clearly stated [5]. In contrast to the history of small antennas, these limitations have been widely taught and appreciated.

The bandwidth of matching, within any specified tolerance of reflection, is proportional to the resonance bandwidth of the resonant circuit. A small bandwidth is logically expressed in terms of the power factor of its reactance, in the manner taught to the writer by Prof. Hazeltine just 50 years ago [1]. Its common expression in terms of $1/Q$ is neither logical nor helpful in clear exposition. The term dissipation factor is numerically equal to power factor but is counter-descriptive of a useful load (as here).

Fig. 2 shows the circuit properties of a small antenna, describing its radiation power factor (PF). The antenna may behave as a capacitor (C) or inductor (L), and either is to be resonated by a reactor of the opposite kind. Dissipation (other than radiation) is here ignored, because it is treated in the earlier paper [2]. The nominal bandwidth of the resonator is the PF (p) times the frequency of resonance, as usual.

Fig. 3 is the bandwidth of matching within any specified tolerance of reflection (ρ) as given in 1948 by Fano [5]. It is graphed in the terms of the present discussion. For each graph, the number of tuned circuits includes the antenna circuit and any that are added for increasing the bandwidth of matching. The added circuits are taken to be free of dissipation. Usually double tuning is used, in which case the added circuit can reduce the reflection coefficient to the square of its value for single tuning.

V. The Radiation Power Factor

The term "radiation power factor" is a natural one introduced by the author in 1947 [2]. It is descriptive of the radiation of real power from a small antenna taking a much larger value of reactive power. It is applicable alike to either kind of reactor and its value is limited by some measure of the size in either kind.

Fig. 4 shows small antennas of both kinds (C and L) occupying equal cylindrical spaces [2]. They are here used for introducing the relation between radiation PF and size.

A small antenna of either kind is basically a reactor with some small value of PF associated with useful radiation. The latter depends primarily on its size relative to the wavelength (λ), as discovered by the writer [2]. The size may be stated relative to the radianlength ($\lambda/2\pi$) in terms of either of two values of reference volume:

$$\text{radiancube} = V_c = \left(\frac{\lambda}{2\pi}\right)^3 = \frac{3}{4\pi} V_s \qquad (1)$$

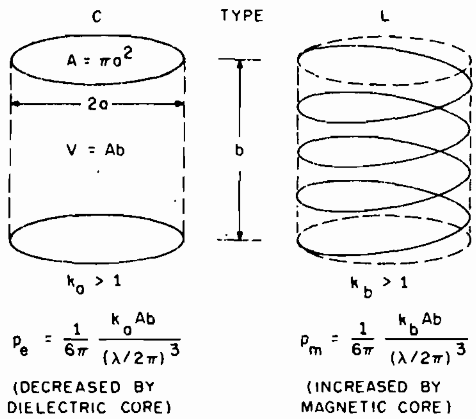

Fig. 4. Radiation power factor in terms of equivalent volume.

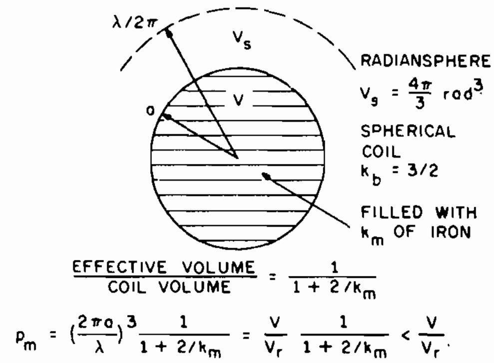

Fig. 5. Spherical coil with magnetic core.

or

$$\text{radiansphere} = V_s = \frac{4\pi}{3}\left(\frac{\lambda}{2\pi}\right)^3 = \frac{4\pi}{3} V_c. \quad (2)$$

The former was used in the writer's first paper. The latter is particularly significant in radiation because it defines the space in which the reactive power density exceeds the radiation power density [10]. Also the latter is convenient if the antenna is spherical [9] or its effective volume is expressed as a sphere.

In either type of antenna, the radiation PF is found to be proportional to volume and also to a shape factor. The cylindrical volume ($V = Ab$) is here multiplied by a shape factor (k_a or $k_b > 1$) to give the effective volume ($V' = k_a Ab$ or $k_b Ab$). Then the general formula is

$$\text{rad PF} = p \approx \frac{1}{6\pi}\frac{V'}{V_c} = \frac{2}{9}\frac{V'}{V_s}. \quad (3)$$

The effective volume may be stated as a sphere of radius (a'), in which case

$$V' = \frac{4\pi}{3}a'^3, \quad p = \frac{2}{9}\left(\frac{2\pi a'}{\lambda}\right)^3, \quad a' = \frac{\lambda}{2\pi}\left(\frac{9}{2}p\right)^{1/3}. \quad (4)$$

It is noted in passing that a certain shape of self-resonant coil radiates equally as both C and L, in which case the total radiation PF is double either one [3].

There is one theoretical case of a small coil which has the greatest radiation PF obtainable within a spherical volume. Fig. 5 shows such a coil and its relation to the radiansphere (V_s) [9], [10]. The effective volume of an empty spherical coil has a shape factor 3/2. Filling with a perfect magnetic core ($k_m = \infty$) multiplies the effective volume by 3:

$$p_m = \frac{2}{9}\frac{(3)(3/2)V}{V_s} = \frac{V}{V_s} = \left(\frac{2\pi a}{\lambda}\right)^3. \quad (5)$$

This is indicated by the shaded sphere (a).

This idealized case depicts the physical meaning of the radiation PF that cannot be exceeded. Outside the sphere occupied by the antenna, there is stored energy or reactive power that conceptually fills the radiansphere [10], but there is none inside the antenna sphere. The reactive power density, which is dominant in the radiation within the radiansphere, is related to the real power density, which is dominant in the radiation outside.

In a rigorous description of the electromagnetic field from a small dipole of either kind, the radiation of power in the far-field is accompanied by stored energy which is mostly located in the near-field (within the radiansphere) [4], [10]. The small spherical inductor in Fig. 5 is conceptually filled with perfect magnetic material, so there is no stored energy inside the sphere. This removes the "avoidable" stored energy, leaving only the "unavoidable" amount outside the inductor but mostly inside the radiansphere. This unavoidable stored energy is what imposes a fundamental limitation on the obtainable radiation PF.

One of the fallacies in some studies has been the provision of dielectric or magnetic material outside of the space occupied by the antenna conductors, without including that material in rating the size of the antenna. The fundamental limitations are based on the size of all the material structure which forms the antenna. Likewise, such material would naturally be included in a practical evaluation of the size. Fig. 5 shows the empty space outside the antenna but inside the radiansphere (V_s) which space is filled with stored energy and therefore reduces the radiation PF of the antenna.

VI. Application to Typical Antennas

The radiation PF may be evaluated for any kind of small antenna. From its value, we may state the effective volume of the antenna, as formulated (4):

$$V' = 6\pi p V_c = \tfrac{9}{2} p V_s, \quad a' = \frac{\lambda}{2\pi}\left(\tfrac{9}{2}p\right)^{1/3}.$$

This is a useful quantity which can be shown on a space drawing. It gives a direct comparison of the bandwidth capability of different structures. It will be shown for C and L antennas of elementary configurations. It will be drawn as a dashed circle the size of the spherical effective volume.

Fig. 6 shows some examples of an electric dipole with a linear axis of symmetry. A thin wire (a) and a thick conical conductor (b) differ greatly in the occupied volume, but much less in effective volume. The latter is influenced most by length and less by the smaller transverse dimensions.

Fig. 6(c) shows a pair of separated discs [2], which is found to approach the greatest effective volume for some shapes within limited length and diameter. However, any

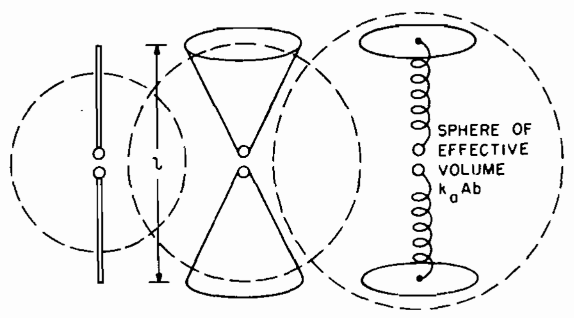

Fig. 6. Effective volume of axial electric dipole.

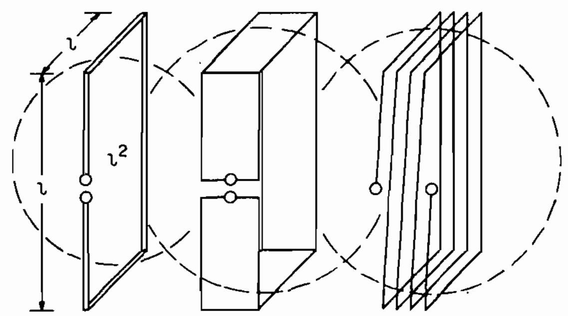

Fig. 7. Effective volume of square loop.

intermediate connecting wires would detract from this rating. The full value of the radiation PF can be realized by the use of a tuning inductor distributed along the axial line between the discs. It is proportioned to conform to the natural pattern of electric potential, thereby contributing no extra amount to the stored electric energy. A coil of small diameter may be used to avoid extra (cross-polarized) radiation therefrom. The spherical effective volume may extend beyond the length between the discs, as shown. This occurs if the disc diameter exceeds $\frac{1}{4}$ the length ($2a > b/4$), as in the example shown. This may be interpreted as a "sphere of influence" extending beyond the antenna structure.

In further reference to Fig. 6(c), there is a pair of end electrodes which will give the greatest radiation PF within a cylindrical boundary. At each end, a hollow cup is connected with its open end toward the center. Its depth is proportioned to maximize the radiation PF. No greater value can be obtained by simple conductors subject to the stated constraints.

Fig. 7 shows some examples of a loop inductor on a square frame. A thin wire (a) and a wide strip (b) differ rather little in effective volume, because it is influenced most by the size of the square. A multiturn loop (c) has nearly the same effective volume as one turn occupying the same space. This is one of the principal conclusions presented in the writer's first paper [2]. It superseded some incomplete evaluations based on the concept of "effective height" of a number of turns, irrespective of their width and spacing.

Referring again to Fig. 4, the shape factors are related to the shape in opposite ways in the two kinds (C and L).

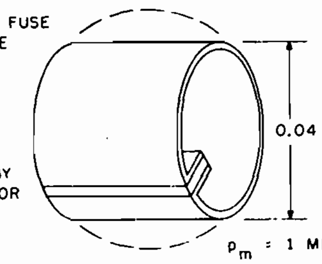

Fig. 8. One-turn loop of wide strip.

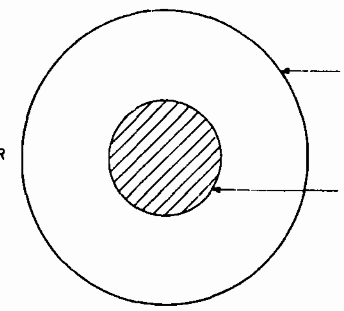

Fig. 9. Radiation shield for use in measuring radiation power factor.

With greater ratio of length/diameter ($b/2a$), one factor (k_a for C) is greater and the other (k_b for L) is smaller. Therefore the utilization of volume is greater for the C type made of a long wire or for the L type made of a "short coil" or loop. These are exemplified in Figs. 6 and 7. Each of these has large and small dimensions, and the smaller dimensions may be less significant in a practical allocation of space.

In the writer's experience, the concept of radiation PF was first applied to the design of a very small loop antenna for coaxial location in the nose cone of a small rocket. Fig. 8 shows the resulting one turn of wide strip. It superseded some attempts to design a multiturn loop. It is resonated by an integral capacitor made of a ceramic slab metallized on both faces. It proved superior in performance, simplicity, and ruggedness. It may have been the smallest antenna then known to realize about 50 percent radiation efficiency, the size being rated in fractions of the wavelength. Its diameter and length were about 0.04 wavelength so its radius was about 0.12 radianlength. It was measured by a method to be described here.

For efficiency of radiation, a small antenna of one kind is resonated by a reactor of the opposite kind. Then

$$\text{radiation efficiency} = \frac{\text{radiation PF}}{\text{rad PF} + \text{loss PF}}. \tag{6}$$

In a very small antenna, the radiation and loss power factors may be so small that their ratio is difficult to measure. In any case, how would they be separated in measurement? Direct measurement of radiated power is laborious. Another method was developed, using a "radiation shield" [10].

Fig. 9 shows the concept of the radiation shield. Its purpose is to avoid radiation of power while leaving the inherent dissipation in the resonant circuit of the small antenna. The shield is a box with conductive walls for preventing radiation. Its size and shape are noncritical,

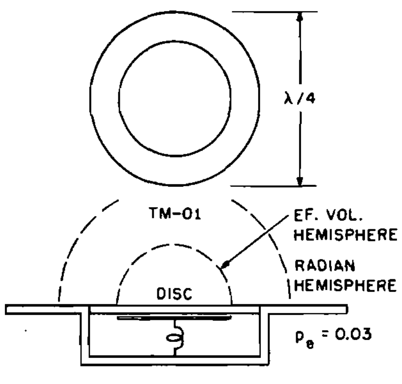

Fig. 10. Flush disc capacitor.

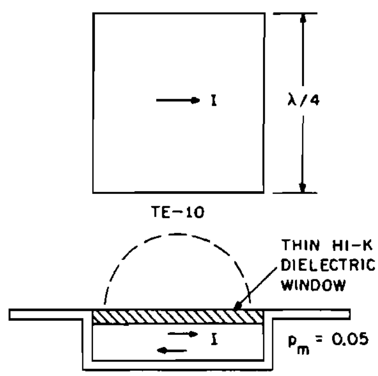

Fig. 11. Flush cavity inductor with dielectric window.

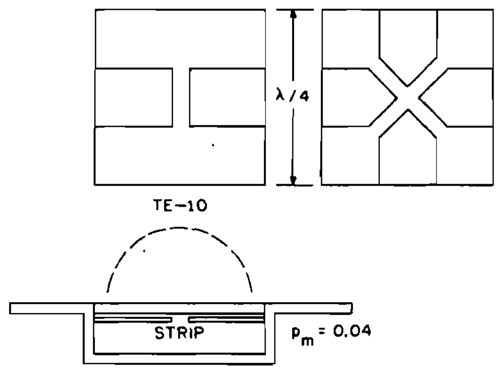

Fig. 12. Flush strip inductor.

but the theoretical ideal is a radiansphere as indicated. It should be much larger than the antenna to be shielded, so as to retain substantially the reactance and loss PF of the antenna. Then the PF is measured with and without the shield, for evaluating the power efficiency of the useful radiation [10]. In the design shown in Fig. 8, the circuit was included in an oscillator, so the effect of the shield on the amplitude of oscillation could be interpreted in terms of radiation efficiency.

VII. Flush Antennas

A useful family of small antennas comprises those that are recessed in a shield surface, such as a ground plane or the skin of an aircraft. Some may be inherently flush designs, while others may be suited for operation adjacent to a shield surface, whether recessed or not. The antenna may be C or L type, either one radiating in a polarization compatible with the shield surface.

Fig. 10 shows a flush disc capacitor. (It is sometimes termed an "annular slot.") This capacitor in the flush mounting may be compared with the same capacitor just above the surface. The recessing somewhat reduces the radiation PF. The remaining effective volume is that of a hemisphere indicated by the dashed semicircle. Its size is comparable with that of the disc. The cylindrical walls may be regarded as a short length of waveguide beyond cutoff, operating in the lowest TM mode (circular TM-01, as shown, or rectangular TM-11). The capacitor may be resonated by an integral inductor as shown. In any cavity, there is a size and shape of disc that can yield the greatest radiation PF. The primary factor is the size of the cavity.

The evaluation of a flush antenna includes the shield surface. It is necessary first to evaluate the radiation PF by some method of computation. Then it can be stated in terms of a volume ratio. Here we consider the half-space of radiation and show the hemisphere of $\frac{1}{2}V'$ which may then be compared with the half-radiansphere, $\frac{1}{2}V_s$. The radii are retained (a' and $\lambda/2\pi$). An antenna located on the surface (not recessed) could be considered with its image to yield the complete sphere of V' to be compared with the radiansphere V_s. Then $\frac{1}{2}$ of each may be shown above the shield plane, as for the flush antenna.

The disc capacitor radiates in the same mode as a small vertical electric dipole, by virtue of vertical electric flux from the disc. This is vertical polarization on the plane of the shield, with omnidirective radiation. The other examples of a flush antenna, to be shown here, radiate as a small horizontal magnetic dipole, by virtue of magnetic flux leaving the cavity on one side and returning on the other side. This is vertical polarization but directive in a figure-eight pattern. Omnidirective radiation can be provided by quadrature excitation of two crossed modes in the same cavity. The radiation PF of either kind is reduced by recessing, but the magnetic dipole suffers less reduction.

Fig. 11 shows an idealized cavity resonator which radiates as an inductor. The cavity is covered by a thin window of high-k dielectric which serves two purposes. It completes the current loop indicated by the arrows (I). Also it provides, in effect, series capacitance which resonates the current loop. The cylindrical walls and the aperture excitation may be regarded as the lowest (cutoff) TE mode (circular TE-11), or rectangular TE-10 or TE-01, as shown). Each of these modes has two crossed orientations, of which one is indicated by the current loop. The continuous dielectric sheet on a square (or circular) cavity resonates the two crossed modes. Because each resonance is in the lowest mode, it involves the smallest amount of stored energy relative to radiated power, and therefore the greatest value of radiation PF.

Fig. 12 shows some practical designs which yield nearly the same performance by the use of conductive strips on ordinary (low-k) dielectric windows. (High-k dielectric is not required.) Here the radiating inductor (strip) and the resonating series capacitor (gap) are apparent. The two

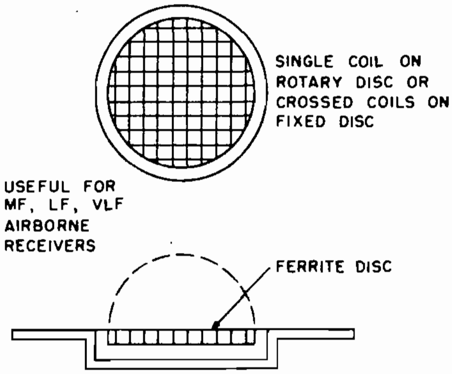

Fig. 13. Flush inductor on thin ferrite disc.

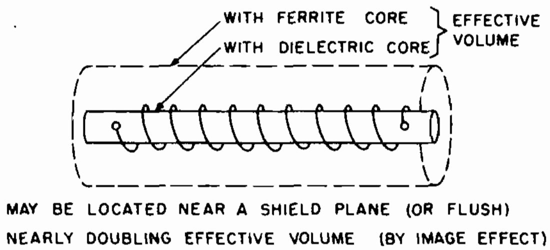

Fig. 14. Long coil on ferrite rod.

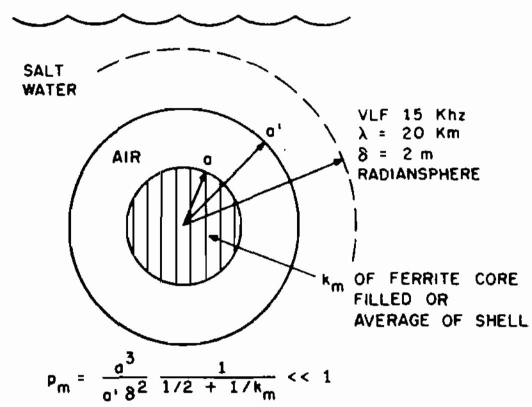

Fig. 15. Inductor in radome submerged in sea water.

alternatives are shown, one mode or a pair of crossed modes. Practical designs about $\lambda/4$ square have been made with radiation PF about 0.04. This is about the largest size that follows the rules of a small antenna.

The required coupling with any of the resonant antennas in Figs. 10–12 may be provided by another (smaller) resonator located within the cavity. This enables the bandwidth of matching shown by the intermediate graph in Fig. 3. Each of these is suited for self-resonance, and requires some depth of activity to hold down the extra amount of energy storage in this nonradiating space.

Fig. 13 shows a flush inductor made of crossed coils on a thin magnetic disc. At medium or low frequencies (MF, LF, VLF) the available ferrite materials [12] can provide a magnetic core which is a return path nearly free of extra energy storage, even in the thin disc; also which adds very little dissipation. The required depth of cavity is then only sufficient to take the disc thickness with some margin. Relative to the wavelength at the lower frequencies, the antenna is too small to enable high efficiency, even at its frequency of resonance, so it is useful only for reception. A rotary coil or crossed coils can be used for a direction finder or omnidirectional reception. The principal application is on the skin of an aircraft.

Fig. 14 shows the ferrite-rod inductor which is the antenna most commonly used in small broadcast receivers (MF, around 1 MHz). The ferrite rod greatly increases the effective volume of a thin coil, as indicated. The effective volume is then determined primarily by the length, rather than the diameter, of the coil. Like the ferrite disc, this can be used close (parallel) to a shield surface or recessed in the surface.

Here we may note that a long coil, with its small shape factor ($k_b \to 1$), can have its effective volume greatly increased by a ferrite core. On the other hand, a parallel-plate capacitor, with its small shape factor ($k_a \to 1$), can only have its effective volume decreased by a dielectric core. This is one respect in which the inductor offers more opportunity in design. In another respect, the number of turns can be used to set the impedance level, a freedom that may be desired but is unavailable in a simple capacitor.

If a long coil as a magnetic dipole were filled with perfect magnetic material, its effective volume would be comparable with that of an equally long conductor as an electric dipole. If the coil had many turns, they could theoretically be distributed (crowded toward the ends) to give an effective volume greater than that of a pair of discs far apart, Fig. 6(a). If the coil is not too thin, this result can be approximated at the lower frequencies with many turns on a ferrite core.

VIII. Antennas for VLF

The greater the wavelength, the more relevant may be the concept of a small antenna. Current activities go as low as 10 kHz with a wavelength of 30 km. Even the largest of transmitter antennas is small in terms of this wavelength, or its radianlength of 5 km. For underwater reception, however, the radianlength or skin depth in salt water is only a few meters, so a small antenna may occupy a substantial fraction of this size. The latter will be discussed first, as another example of a small inductor.

For submarine reception of VLF signals in salt water, an inductor in a hollow cavity (radome) is the preferred type [8]. As compared with a capacitor, its efficiency is greater because the conductivity of the water causes near-field losses in response to electric field but not magnetic field. Also there is no need for conductive contact with the water.

Fig. 15 shows an idealized small antenna in a submarine cavity [8], [9]. It is a spherical coil with a magnetic core, as shown in Fig. 5. In the water, the radianlength is equal to the skin depth (δ). At 15 kHz, this is about 2 m. The size of the cavity is much less, and the coil still less, so it is a small antenna in this environment. The radiation PF indicates two qualities, the desired coupling to the medium and the undesired dissipation in the medium. The former is proportional to the coil volume, and is increased by the magnetic core. The latter is decreased by increasing the

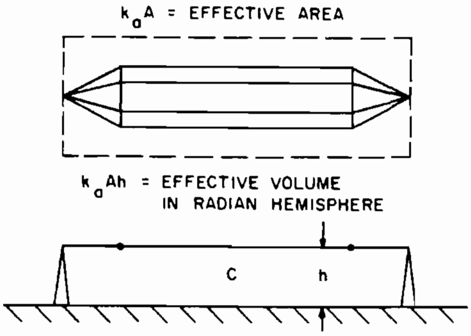

Fig. 16. Large flat-top capacitor which is still small relative to wavelength.

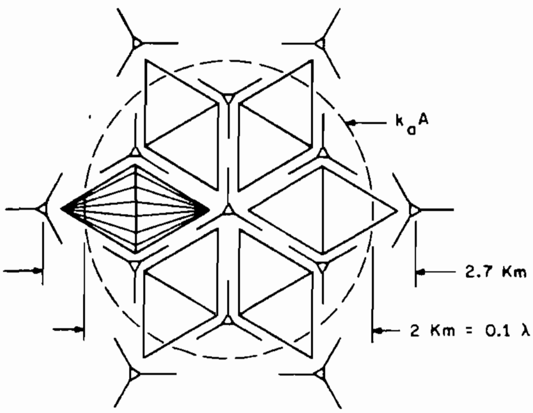

Fig. 17. Large VLF antenna (plan view).

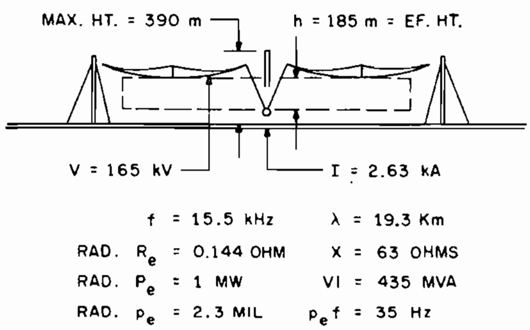

Fig. 18. Large VLF antenna (elevation view).

cavity radius. The coil is in the vertical plane for vertical polarization. Crossed coils may be used for omnidirective reception and direction finding.

For efficient transmission at the lower frequencies, one of the early simple types is the one shown in Fig. 16 [7]. It is a "flat-top" grid of wires forming a capacitor with ground as the lower conductor. In the terms of small antennas, it may be described in the manner indicated. The effective height (h) is related to the radiation resistance. The capacitance enables the statement of an effective area (k_aA) as noted. The effective volume (k_aAh) in half-space is compared with $\frac{1}{2}$ radiansphere to determine the radiation PF. It is notable that the grid of many wires may provide an effective area greater than that of the grid, in spite of the much smaller area of conductor.

As an extreme example, we shall consider the later one of the two largest antennas in the world. They are the Navy transmitters located at Cutler, Me., (NAA) and Northwest Cape, Australia, (NWC). The latter was commissioned in 1967. It is taken as an example because it is the simpler. Figs. 17 and 18 show the plan and elevation views of the structure. It operates down to about 15 kHz, a wavelength of 20 km.

The lowest "specification" frequency determines the required size. At this frequency, the following statistics are relevant:

frequency	15.5 kHz
wavelength	$\lambda = 19.3$ km
extreme diameter	2.7 km $= \frac{1}{7}\lambda$
center-tower height	390 m
effective height	185 m $= \frac{1}{104}\lambda$
capacitance	0.163 μF
effective area	3.4 (km)^2
effective volume	$V' = 0.63$ (km)^3
radiation resistance	$R_e = 0.144\ \Omega$
reactance	$X_e = 63\ \Omega$
radiation PF	$p_e = 2.3$ mils
loss PF	$<$ 2.3 mils
efficiency	$>$ 0.50
resonance bandwidth	134 Hz
radiated power	1 MW
input power	2 MW
reactive power	435 MVA
voltage	165 kV
current	2.63 kA.

Particularly spectacular are the reactive power of 435 MVA in the air dielectric, and the real power of 2 MW delivered to a resistance of about 0.3 Ω. Less than half of this resistance is budgeted to all losses, including the ground connection and the tuning inductor. The small value of radiation PF (2.3 mils) well qualifies this structure as a "small antenna." The choice of a capacitor (rather than an inductor) was influenced by the need for omnidirective coverage.

The effective volume is diagramed in the form of a cylinder bounded by the dashed lines. Fig. 17, the effective area is a circle including more area than the grid of wires. In Fig. 18, the effective height is reduced by two practical considerations. The top level is lower than the top wires by the effect of the downleads (48 wires around the central tower). The bottom level is higher than the ground, by the effect of the grounded towers and guy wires (each tower having 3 at each of 4 or 5 levels). The resulting effective height is about $\frac{1}{2}$ the average height of the 13 towers. The radiation PF is related to this effective volume by (3) adapted to half-space above ground. (The effective volume is compared with $\frac{1}{2}$ radiansphere.)

IX. Conclusion

The principles of small antennas can be described in simple terms, both mathematically and pictorially. They are helpful in the understanding and design of practical antennas in either type, capacitor or inductor. While the two types have a common rating in terms of effective volume, there are differences that may give either an advantage in size or other practical considerations. For any configuration, the efficiency and/or bandwidth is ultimately limited by size relative to the wavelength.

Acknowledgment

By way of acknowledgment, the author is indebted to various groups for opportunities to apply the principles of small antennas. The first example was the one-turn loop of Fig. 8 which was developed in collaboration with Seymour Berkoff at Emerson Radio and Phonograph Corp., in 1948 (for use by NBS in a proximity fuse). In the design of the large VLF antennas for the Navy by DECO at Leesburg, Va. (now DECO Communications Dep. of Westinghouse), the author was active in 1956–1957 as consultant to the late Lester H. Carr and his group, including William S. Alberts who kindly provided the information for Figs. 17 and 18. The flush inductor of Fig. 12 was developed in several forms at Wheeler Laboratories during the period 1964–1970, for use on rockets and aircraft. The work was supported by various agencies, including Bell Telephone Laboratories (for Army Ordnance), Air Systems Division of the Air Force, and Naval Air Development Center. Other examples in the text are based on specific studies and proposals made in various situations during the past quarter century. The writer is grateful to his associate, Alfred R. Lopez, for helpful discussions relating to this paper.

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FUNDAMENTAL RELATIONS IN THE DESIGN OF A VLF TRANSMITTING ANTENNA

FUNDAMENTAL LIMITATIONS OF A SMALL VLF ANTENNA FOR SUBMARINES

BY
HAROLD A. WHEELER

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Fundamental Relations in the Design of a VLF Transmitting Antenna*

HAROLD A. WHEELER†

Summary—**For a VLF flat-top antenna much smaller than the radian sphere (a sphere whose radius is one radianlength), the effective height, effective area, and effective volume are defined. The required power factor of radiation proportionately determines the effective volume. For a specified power to be radiated, the effective height inversely determines the current and the effective area inversely determines the voltage. For a limited electric gradient on the overhead wires, the current requires a proportionate area of conductor surface. A corresponding total length of wire in the flat top is adequate if disposed for uniform distribution of charge and if spread out to realize the required effective area. These objectives are obtained more readily by some configurations, such as long parallel wires or concentric circles of wire. This study has been made for the U. S. Navy's high-power transmitter to be located in Maine, the first to radiate 1 megawatt continuously at 15 kc.**

THE antenna for a high-power VLF transmitter is a large structure. It is important to know just what size is needed to meet any particular requirements. Therefore, concepts and formulas will be given for the fundamental relations that govern the design of such an antenna.

The principal objective is to radiate a specified amount of power over a sufficient bandwidth of frequency. Efficient radiation requires that the bandwidth be provided largely by the radiation power factor, to which is added the power factor of all losses in the antenna circuit. For example, an efficiency of 0.50 requires that one half the bandwidth be provided by the radiation power factor.

The performance capabilities of the antenna are limited by its size and such other factors as the current for corona on the wires and the voltage of insulation. These will be formulated.

Fig. 1 outlines the type of antenna, having a flat-top web of wires suspended at a height above the ground. The antenna is assumed to be much smaller than the radian hemisphere (a hemisphere whose radius is one radianlength), behaving as a pure capacitance with negligible inductance. The relations are valid for single or multiple tuning; in the latter case, all the tuning circuits are regarded as connected in parallel to form a single circuit.

Some detrimental factors are included implicitly, such as the reduction of the effective height by the proximity of grounded supporting towers. Ground losses and other losses are omitted because they are outside the fundamental relations.

Since one of the principal limitations is imposed by corona on the wires, this topic is presented first as back-

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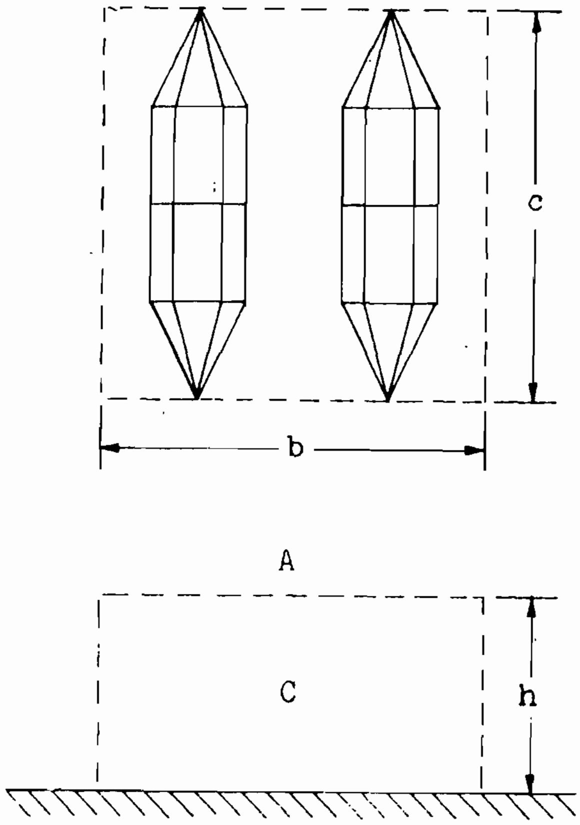

Fig. 1—Essential dimensions of flat-top antenna.
C = capacitance.
h = effective height.
$A = bc$ = effective area.
A_a = conductor area (of surface of wires).

ground. In air as a dielectric or insulator, there is a fairly definite value of alternating-voltage gradient which is just sufficient to cause breakdown and sparking in a uniform electric field. In the vicinity of round wires far from other objects, a somewhat greater gradient is permissible on the conductor surface, because the gradient decreases with increasing distance from the surface. When the permissible gradient is exceeded on the surface, the wire is surrounded by a corona discharge rather than sparking.

An empirical formula has been published for evaluating the corona gradient on a round wire:[1]

$$E_c = E_b(1 + \sqrt{a_1/a}) \qquad (1)$$

in which

E_c = voltage gradient for corona on round wire (rms kv/mm),
E_b = 2.05 kv/mm = voltage gradient for breakdown in uniform field (rms),
a_1 = 0.90 mm = wire radius (mm) on which $E_c = 2E_b$,
a = wire radius (mm).

The most favorable condition is a uniform distribution of charge, and hence a uniform gradient, all over the conductor surface of the wires. To obtain a reasonable value of gradient for such a design, E_b is multiplied by several factors as follows:

[1] S. S. Attwood, "Electric and Magnetic Fields," John Wiley & Sons, Inc., New York, N. Y., 3rd ed.; 1949. (See p. 91, Peck's formula for extra gradient on round wire for sparkover.)

1.27 for wires of diameter 1 inch, by (1) above.
1/2 for water drops in wet weather (approximate, based on some experience).
1/2 for doubtful factors, including departure from constant gradient.

The resulting value of gradient is

E_a = 0.65 kv/mm (rms).

An example will be carried along with the formulas, based on these specifications:

f = 15 kc = frequency,
λ = 20 km = wavelength,
p = 0.002 = radiation power factor,
P = 1 megw = radiated power,
V = 200 kv = antenna voltage (rms),
E_a = 0.65 kv/mm = gradient on wires (rms), and
R_c = 377 ohms = wave resistance in air.

For radiation efficiency of 0.50, the total power factor would be 0.004; this fraction of the frequency gives a bandwidth of 60 cycles between the points of half-power response.

Several dimensions of the antenna are defined on an idealized basis with reference to Fig. 1.

The *effective height* (h) is the average height of the charge on the antenna and downleads, realtive to the average height of the opposite charge on the ground and supporting towers.[2] This is a statement of the accepted meaning. It is shown as the actual height in Fig. 1, but in practice the effective height is somewhat less. It determines the radiation resistance.

The *effective area* (A) is the area of an idealized parallel-plate condenser, with plates separated by the effective height, which would have the same capacitance as the antenna.[2,3]

The *capacitance* (C) establishes the relation between the preceding pair of dimensions, by the basic formula,

$$C = \epsilon_0 A/h \qquad \text{farads (2)}$$

in which

C = capacitance (farads),
ϵ_0 = electrivity (electric permittivity) of air (farads/meter)
A = effective area (meter^2),
h = effective height (meters).

This is the capacitance in a uniform field bounded by the rectangular prism shown in dotted lines in Fig. 1.

The *conductor area* (A_a) is the area of the surface of all conductors forming the antenna and downleads. It is much smaller than the effective area. The conductor area, if uniformly charged, is capable of holding an amount of charge that is limited by corona. This charge

[2] This parallel usage of "effective height" and "effective area," as defined here, should not be confused with other usages of these same terms.

[3] J. T. Bolljahn and R. F. Reese, "Electrically small antennas and the aircraft antenna problem," IRE Trans., vol. AP-1, pp. 46–54; October, 1953. (Equivalent area defined, same as effective area in present paper; no reference to voltage limitation for radiation power.)

is proportional to the antenna current, regardless of the antenna voltage.

The first relation to be expressed is one which influences the frequency bandwidth and the radiation efficiency, but is independent of the amount of power.

The *effective volume* (Ah) is proportional to the required value of the *radiation power factor* (p).[4]

$$Ah = \frac{3p\lambda^3}{8\pi^2} \quad \text{meter}^3 \quad (3)$$

$$= 0.608 \text{ km}^3$$

in which

Ah = effective volume (meter^3),
p = radiation power factor = ratio of radiation resistance/reactance, and
λ = wavelength (meters).

The radiation power factor contributes to the total power factor which proportionately determines the frequency bandwidth. The radiation efficiency is the ratio of radiation power factor over total power factor.

The next pair of relations are determined by the amount of power to be radiated.

$$hI = \frac{\lambda}{2\pi}\sqrt{\frac{3\pi P}{R_c}} = \frac{\lambda}{4\pi}\sqrt{\frac{P}{10}} \quad \text{meter-amperes} \quad (4)$$

$$= 0.503 \text{ km-ka.}$$

$$AV = \left(\frac{\lambda}{2\pi}\right)^2\sqrt{3\pi P R_c} = \frac{3\lambda^2}{2\pi}\sqrt{10P} \quad \text{meters}^2 \text{ volts} \quad (5)$$

$$= 604 \text{ km}^2\text{kv}$$

in which

I = antenna current (rms amperes),
V = antenna voltage (rms volts),
P = radiated power (watts), and
R_c = 377 ohms = wave resistance in air (ohms).

The former of these relations (meter-amperes) has been used as a rating of the radiated power. The latter (meter^2 volts) seems to be a new concept; it expresses an inverse proportionality between effective area and voltage.

The remaining relation involves the power and the average electric gradient on the conductor surface.

$$A_a h = \frac{\lambda^2}{4\pi^2 E_a}\sqrt{3\pi P R_c} = \frac{3\lambda^2}{2\pi E_a}\sqrt{10P} \quad \text{meters}^3 \quad (6)$$

$$= 930{,}000 \text{ m}^3$$

in which

A_a = conductor area (meters^2)
E_a = average electric gradient on conductor surface (rms volts/meter).

This expresses an inverse relationship between the conductor area and the effective height.

These four relations will be applied here to determine the dimensions for the stated example.

(5) $A = (AV)/V = 3.02$ km^2,
(3) $h = (Ah)/A = 0.200$ km $= 200$ m,
(6) $A_a = (A_a h)/h = 4650$ m^2.

For wire of 1-inch diameter, the required length of wire is 58 km. The wire must be distributed to give the required effective area, which is usually comparable with the actual area occupied by the flat-top pattern of wires. One of the most critical problems is the pattern of distribution of the wire to assure nearly constant gradient on nearly all of the conductor surface. Economy of conductor area is dictated by its proportionality to the wind loading or the ice-melting power, and approximately to the ice loading. For a certain area, the former is nearly independent of wire size and length, while the latter is substantially greater for smaller wires. The effective area is more dependent on the number of wires, so it is greater for a greater length of smaller wire; this rule is applicable where icing is not encountered.

Some other properties of this example are the following:

$R = 0.160$ ohm = radiation resistance,
$X = R/p = 80$ ohms = reactance,
$C = 0.133$ μf = capacitance,
$I = V/X = 2.5$ ka = antenna current.

The pattern of wires forming the flat top and downleads presents an opportunity for much ingenuity in design and approximate computation. Fig. 1 shows an array of parallel wires, termed the "triatic" after the nautical term for the supporting catenaries across the array. This form offers an inherent tendency for uniform charge distribution, and also the simplest formulas for approximate computation. The two separate structures are intended to be connected together for normal operation, or either half operated independently at reduced power while the other half is out of service.

The relations given here are most useful for estimating the size of antenna needed for realizing a specified performance. They are also useful for evaluating the capabilities of an existing antenna. The emphasis has been placed on the simplest concepts relating the antenna structure with its operating requirements.

This study has been made in connection with the design of the U. S. Navy's high-power transmitter to be located in Maine. The computed example gives some idea of the size of antenna required to radiate 1 megawatt at 15 kc.

[4] H. A. Wheeler, "Fundamental limitations of small antennas," Proc. IRE, vol. 35, pp. 1479–1484; December, 1947. (Antenna smaller than the radian sphere.)
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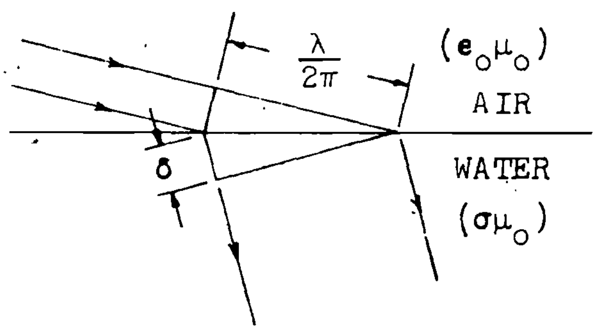
Fundamental Limitations of a Small VLF Antenna for Submarines*

HAROLD A. WHEELER†

Summary—**A submarine requires a small VLF antenna for reception while submerged. Since the propagation in sea water is nearly vertical (downward from the surface), the only operative types are horizontal dipoles, electric and magnetic. The electric dipole is coupled by conduction and the magnetic dipole by induction in a loop. The former has no resonance and nearly unlimited bandwidth, but fails when not submerged. The latter, by resonance, is able to present much greater interception area and available power. The magnetic interception area is determined by the size of the radome and by the radianlength or skin depth in sea water (2 meters at 15 kc). The radiation power factor, which is essential to bandwidth and efficiency, is influenced also by the size of the inductor and by the magnetic permeability of an iron core. Simple formulas illustrate these relations for the idealized spherical shape of radome, coil and core. Omnidirectivity in azimuth requires crossed coils in a two-phase circuit.**

THE fundamental limitations of a small antenna have been fairly well stated for a location above ground, but seem to have been neglected for a location under the surface of sea water. The latter is the problem of the small antenna mounted on a submarine for reception of VLF signals while submerged.

The propagation of a radio wave over a water surface is well understood. For present purposes, it will be summarized with reference to Fig. 1. Just above the



Skin depth: $\delta = \frac{1}{2\pi}\sqrt{\frac{\lambda}{30\sigma}}$

Fig. 1—Wave refraction at water surface.

surface, the direction of propagation is tilted downward and a fraction of the power is directed into the water. The index of refraction, as determined by the conductivity of the water, is very great, so the direction of propagation in the water is nearly vertical.

The propagation in the water is greatly attenuated by the skin effect which prevails in conductors. The *skin depth* or *napier depth* is defined as the depth at which the fields are attenuated one napier (8.7 db) below their value at the surface.

$$\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}} = \sqrt{\frac{\lambda}{\pi R_c\sigma}} = \frac{1}{2\pi}\sqrt{\frac{\lambda}{30\,\sigma}} \tag{1}$$

δ = skin depth, napier depth, or radianlength in water (meters).
$\omega = 2\pi f$ = radian frequency (radians/second).
$\mu_0 = 0.4\,\pi\ 10^{-6}$ = magnetivity (permeability) in air or water (henries/meter).
σ = conductivity in water (mhos/meter).
λ = wavelength in air (meters).
$\lambda/2\pi$ = radianlength in air (meters).
$R_c = 120\pi$ = wave resistance in air (ohms).

The same distance (δ) is also the *radianlength* in the water, so the index of refraction is $\lambda/2\pi\delta$ as diagrammed in Fig. 1.

The conductivity of sea water is about 4 mhos/meter. The lowest frequency in VLF service is about 15 kc, at which the skin depth is 2.0 meters. The index of refraction is 1600, so the direction of propagation in the water is practically vertical. The attenuation with depth is one napier per 2.0 meters, or 4.3 db/meter.

Vertical propagation downward in the water is accomplished by horizontal crossed electric and magnetic fields. The orientation of these fields in azimuth is determined by the direction of the wave over the surface. There is practically no pickup by a submerged vertical dipole of either kind, so there is no simple antenna for omnidirective reception corresponding to the vertical electric dipole above the surface. Omnidirective reception under the water is obtainable by a pair of crossed horizontal dipoles coupled to the receiver in phase quadrature. The further discussion is directed to a single horizontal dipole, submerged to a depth greater than the skin depth in the water.

The horizontal magnetic dipole is realized by a coil or loop whose axis is horizontal. Fig. 2 shows a spherical coil of radius a in the center of a spherical radome of larger radius a'. The coil is filled with a core of magnetic ratio k. The radome is the boundary of insulating material between the water outside and air inside. It is assumed much smaller than the radian sphere of radius δ in the water. Following the principles that are recognized for small antennas in free space, we shall formulate the principal properties of the submerged coil antenna.

The *radiation power factor* of the coil determines its capabilities as an efficient radiator over a frequency bandwidth. It is a measure of the power coupled to the surrounding water, as compared with the reactive power in the inductance.

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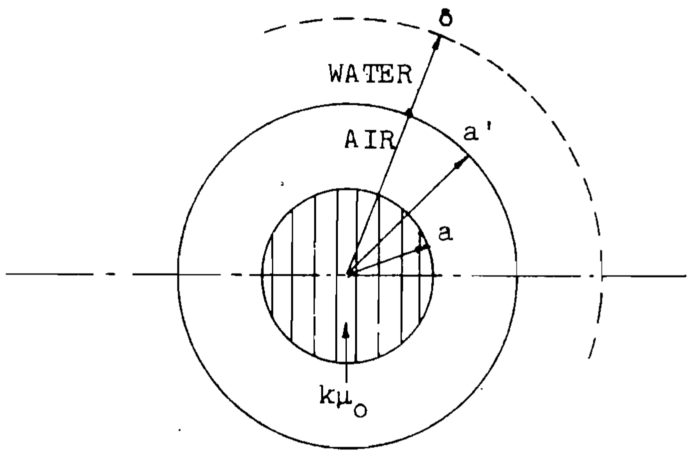

Fig. 2—Coil in submerged radome.

$$p = \frac{a^3}{a'\delta^2}\frac{1}{1/k + 1/2} \ll 1 \qquad (2)$$

in which
p = radiation power factor,
a = radius of spherical coil,
a' = radius of spherical radome,
k = magnetic ratio of core in coil.

As in the case of a coil in free space, this power factor is proportional to the volume of the coil and is less for greater radianlength. In this case it also is less for greater size of radome. A greater magnetic ratio causes lesser magnetic energy in the coil and thereby increases the power factor.

The *interception area* of the coil is the usual concept for formulating its ability to take some power from the wave and make it available to a load. This area is here defined with reference to the power density in the air wave over the surface, so it can be compared with the same rating for a coil in the air.

$$A_m = \frac{3\pi^2 a'\delta^2}{2\lambda} \exp - \frac{2d}{\delta} \qquad (3)$$

in which
A_m = interception area of submerged magnetic dipole,
d = depth of dipole from surface.

The first factor is the area before the attenuation by depth, and the second factor is that attenuation.

The available power, as a basis for defining the interception area, is the power that the antenna could deliver to an ideal matched load, by tuning out the coil reactance and matching the radiation resistance. As in the case of a coil above the surface, the interception area is determined by the environment of the coil and is independent of the properties of the coil itself. This area increases with the size of the radome. The first factor is independent of the wavelength, since this dimension appears in both numerator (implicitly in δ^2) and denominator.

A spherical coil as large as the radome, with a perfect magnetic core, is the ideal for fully utilizing the volume of the enclosed air space to obtain the greatest power factor of radiation. This would place a lower limit on the size of radome required to realize a certain power factor of radiation. If the radome is larger, the coil may be smaller, may be of different shape, and may dispense with the iron core. For a submarine, it is more important to reduce the size of the radome than to economize in the weight of the coil and core.

The alternative kind of antenna is the horizontal electric dipole, coupled with the water by conduction. It is essentially a pair of electrodes having their outer surfaces in contact with the water (directly or through thin dielectric walls of a radome). For comparison in the same spherical radome, the electric dipole is adapted to a spherical space; its electrodes are spherical caps of such size as to give nearly uniform field inside the sphere.

Unlike the coil, the electric dipole has the negligible reactance of short connecting wires. Its radiation resistance is that of the water outside the radome, which is of the order of 1 ohm. Therefore, the radiation power factor is practically unity and there is no difficulty coupling to a resonant circuit of any usual bandwidth.

The matched load for a spherical radome is a resistance whose value is one half as great as that of the same sphere filled with the same water. This is because the external resistance of a sphere is one half as great as its internal resistance if the mediums are the same. Such a matched load is assumed for evaluating the available power.

The interception area of the electric dipole is evaluated on the same basis.

$$A_e = \frac{3\pi^2 a'^3}{2\lambda} \exp - \frac{2d}{\delta} \qquad (4)$$

in which

A_e = interception area of submerged electric dipole.

The first factor is dependent on the size of radome and the wavelength but not on the conductivity of the water (involved in the skin depth).

On the present assumption of a radome much smaller than the radian sphere, the magnetic dipole or coil is much superior in its interception area.

$$A_m/A_e = (\delta/a')^2 \gg 1 \qquad (5)$$

This ratio of superiority is directly proportional to the wavelength. As an example, take a radome of radius $a' = 0.30$ meter or 1 foot; in sea water at 15 kc, this ratio of area or intercepted power is $(2.0/0.30)^2 = 44$ (or 16.5 db).

In practice, the horizontal dipole may be rotated for maximum pickup and for direction finding. Here the coil has the advantage of convenience in rotation, since the coil does not have to be very close to the wall of the radome.

Since the magnetic properties of air and water are similar, and the magnetic field is horizontal in both, the coil will operate nearly the same when above or below the surface. On the other hand, the conduction properties are opposite, and the electric fields are respectively vertical and horizontal, so the same electric dipole cannot operate in both mediums.

It is concluded that the magnetic dipole or coil antenna is the preferred kind for use on a submarine, and has a great advantage if limited to the interior of a small radome. Its radiation power factor and interception area are defined and formulated as representing its most significant ratings of performance.

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The Radiansphere Around a Small Antenna*

HAROLD A. WHEELER†, FELLOW, IRE

Summary—The "radiansphere" is the boundary between the near field and the far field of a small antenna. Its radius is one radianlength ($\lambda/2\pi$), at which distance the three terms of the field are equal in magnitude. A "small" antenna is one somewhat smaller than the radiansphere, but it has a "sphere of influence" occupying the radiansphere. The power that theoretically can be intercepted by a hypothetical isotropic antenna is that which flows through the radiansphere or its cross section, the "radiancircle."

From a small electric dipole, the far field of radiation is identified as a retarded magnetic field. Between two such dipoles, the far mutual impedance is that of mutual inductance, expressed in terms of space properties and the radiansphere.

A small coil wound on a perfect spherical magnetic core is conceived as an ideal small antenna. Its radiation power factor is equal to the ratio of its volume over that of the radiansphere. A fraction of this ratio is obtainable in various forms of small antennas (C or L) occupying a comparable amount of space.

A radiation shield, in the form of a conducting shell the size of the radiansphere, enables separate measurement of radiation resistance and loss resistance.

Introduction

The subject of small antennas deals with the problems of effective radiation and interception by structures whose dimensions are much less than one wavelength. This assumption of small size reduces to simplest terms the antenna properties and the resulting limitations in practical applications. The concepts and rules to be presented are readily appreciated and easily retained for future reference.

The scope of this paper is limited to some principles and viewpoints that are elementary but have not previously been integrated and clearly presented. They come from various sources and have been assembled by the writer in the course of occasional studies and design experience for widely diversified purposes over the past 15 years or so.

Several concepts appear to have been original with the writer, although based on well-known principles. The "radiansphere" is developed to describe the boundary of the transition between near field and far field, and is given significance as the "sphere of influence." The "radiancircle" is the interception area of the hypothetical isotropic radiator. The "radiation power factor," previously introduced by the writer, is formulated for an idealized spherical antenna much smaller than the radiansphere. The "radiation shield," a spherical conductor located at the radian sphere, is presented to enable separate determination of radiation resistance and loss resistance, hence the radiation efficiency. The mutual impedance between small dipoles is simple and useful but seldom stated; here it is analyzed into the three kinds of impedance components (C, R, L), and is formulated directly in terms of the mean radiation resistance of the sending and receiving antennas.

After a list of symbols, the presentation will start with a brief reference to each principal concept, stated in the terminology to be used here.

Symbols

(MKS units: meters, seconds, watts, volts, amperes, ohms, henries, farads.)

l = length of small dipole ($l \ll \lambda/2\pi$)($l \ll r$)
r = radial distance ($r \gg l$)
h = height above plane
a = radius of sphere (inductor)
A = area of small loop
A = interception area of antenna
V = volume (of sphere)
λ = wavelength
$\lambda/2\pi$ = radianlength
f = cycle frequency
$\omega = 2\pi f$ = radian frequency
Z = impedance (complex)
R = resistance (radiation)
L = inductance
C = capacitance
I = current
V = voltage
E = electric field
H = magnetic field
P_1 = power radiated from sending antenna
P_2 = power available from receiving antenna
$R_0 = 377$ = wave resistance of square area of plane wave in free space
μ_0 = magnetivity in free space
ϵ_0 = electrivity in free space
k_m = magnetic ratio (in core of inductor)
n = number of turns (in coil of inductor)
$p = R/\omega L$ = power factor (radiation)
g = power ratio of directivity
sub-a = inductor sphere
sub-r = radian sphere
sub-1, 2 = sending, receiving (antennas)
sub-12 = mutual (between antennas)
* = subject to retardation by distance angle

Basic Concepts

Radiansphere

The radiansphere is a hypothetical sphere having a radius of one radianlength from the center of an antenna

* Original manuscript received by the IRE, December 23, 1958; revised manuscript received, April 14, 1959. This topic has been presented to meetings of graduate seminars in electrical engineering at Johns Hopkins Univ., Baltimore, Md.; December 2, 1954; and at Polytech. Inst. of Brooklyn, Brooklyn, N. Y.; October 12, 1956.
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much smaller than the sphere. Physically, it marks the transition between the "near field" inside and the "far field" outside. While sending, the radiation field comprises stored energy and radiating power, the former predominating in the near field and the latter in the far field. The radiansphere is a measure of the "sphere of influence" of the antenna. It is a convenient reference for all radial distances.

Radiancircle

The radiancircle is the projection of the radiansphere and, conceived as such, is the interception area of the hypothetical isotropic radiator (to be defined) [2].

Radianlength

The radianlength is $1/2\pi$ wavelength (denoted $\lambda/2\pi$) which appears in many formulas for antennas and waves. Its principal significance is its role as the radius of the radiansphere and radiancircle. Any length dimension (l) may be eppressed in terms of its ratio over the radianlength ($2\pi l/\lambda$).

Wave Resistance

The wave resistance of free space ($R_0=120\pi=377$ ohms) is the apparent resistance (V/I or E/H) of a square area of a plane wave in free space. It may be included in any impedance formula to provide the required dimension (ohms) in a significant and convenient form. For example, the reactance of an inductor usually includes the factor $\omega\mu_0$, for which may be substituted $R_0(2\pi/\lambda)$; the latter more directly provides the same dimensions, ohms per meter [5].

Small Antenna

A "small" antenna is one which is much smaller than the radiansphere. Conversely, it is one operating at a frequency so low that its sphere of influence is much greater than its size. It is characterized by a small power factor of radiation, meaning that its radiation resistance is much less than the principal component of its self-reactance. A small antenna is usually a simple electric or magnetic dipole. The near field depends on which kind of dipole, while the far field is the same for either kind. The electric dipole is a current element physically realizable, while the magnetic dipole is a flux element simulated by a current loop [5], [6].

Mutual Impedance

Fig. 1 shows the definition of the complex mutual impedance (Z_{12}) between two electric dipoles (as examples of small antennas). It includes the attenuation of amplitude and the retardation of angle with the distance from sending antenna to receiving antenna [2], [3].

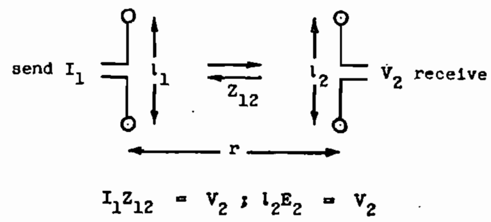

Fig. 1—Definition of mutual impedance between two small electric dipoles.

Efficiency

This is here defined as the maximum efficiency of transmission from a first antenna to a second. It is equal to the ratio of the power available from the second over the power input to the first (P_2/P_1). For present purposes, the associated connection circuits are assumed to be free of dissipation. The second antenna delivers the available power if its resistance is matched to a load resistance while tuning out the reactance of both. If the antennas are separated far enough to give low efficiency, the efficiency may be expressed simply in terms of the radiation resistance of both antennas and the magnitude of mutual impedance therebetween (R_1, R_2, $|Z_{12}|$) [1], [2], [4].

Isotrope

The isotropic radiator or isotrope is one which is conceived to radiate the same in all directions over the sphere in space. It is physically realizable in longitudinal waves (such as sound) but not in transverse waves (such as radio). In any case, it is a helpful concept as a reference for evaluating directivity [2], [4].

Directivity

The usual antennas concentrate their radiated power in some part of the sphere in space. In the direction of greatest concentration, the power ratio of directivity (g) has its maximum value (greater than unity). Inversely, we may say that effectively $1/g$ of the sphere is filled with radiation. The doughnut pattern of a small dipole fills 2/3 of the sphere, so $g=3/2$ [2], [4].

Electric Dipole

The electric dipole is one that radiates by virtue of a current flowing in a length of conductor and returning through the capacitance in the surrounding space. By reciprocity, when exposed to an electric field, it receives an induced voltage proportional to its length. It is the simplest type of radiator for theoretical analysis.

Magnetic Dipole

The magnetic dipole is one that radiates by virtue of magnetic flux from the dipole returning through the surrounding space. It is realized by current in a coil of conductor having a certain total area of coaxial turns. It is distinguished from the electric dipole in that the current returns in the conductor and not in the space capacitance. Its radiation may be computed by regarding each small element of conductor as an electric dipole. Some, but not all, of the general properties to be stated for the electric dipole are valid also for the magnetic dipole.

Far Component of Mutual Impedance

At radial distances much greater than the radianlength, the dominant component of mutual impedance is the one caused by the far field of radiation. Its magnitude may be computed from the mutual inductance between current elements. In doing this, we consider only the magnetic field, ignoring the electric field. The only deficiency is the absence of the retardation caused by the interaction of both fields.

Referring to Fig. 1, the mutual inductance between the two short current elements is given by the Neumann formula (Ramo-Whinnery), [9]:

$$L_{12} = \mu_0 \frac{l_1 l_2}{4\pi r}. \tag{1}$$

From this is computed the mutual impedance, expressed in terms of wave quantities:

$$|Z_{12}| = \omega L_{12} = \frac{R_0}{4\pi} \frac{l_1 l_2}{r(\lambda/2\pi)} = R_0 \frac{l_1 l_2}{2r\lambda} = 60\pi \frac{l_1 l_2}{r\lambda}. \tag{2}$$

The 4π in the denominator appears when formulating a spherical problem in terms of rationalized (cylindrical) units. It is notable that all length dimensions appear in ratios, while the impedance dimension is provided by R_0.

The phase angle of inductive reactance and the retardation by distance are easily added to this formula, as will be shown below in a complete formula.

In radial directions different from Fig. 1, the magnetic-field coupling is opposed in some degree by electric-field coupling to give the characteristic doughnut pattern.

Radiation Resistance

Since the radiation resistance is determined by the radiated power in the far field, it can be computed from the simple formula for mutual impedance. We use also the concept that the doughnut pattern fills only $\frac{2}{3}$ of the sphere. The radiation field is

$$|E_2| = \frac{V_2}{l_2} = \frac{|Z_{12} I_1|}{l_2} = \frac{R_0}{4\pi} \frac{l_1}{r(\lambda/2\pi)} |I_1|. \tag{3}$$

The radiated power, which determines the radiation resistance (R_1), is computed as the product of $\frac{2}{3}$ the area of the distance sphere times the power density of radiation outward through this sphere.

$$P_1 = R_1 |I_1|^2 = \frac{2}{3}(4\pi r^2)|E_2|^2/R_0$$
$$= \frac{2}{3}\frac{R_0}{4\pi}\left(\frac{2\pi l_1}{\lambda}\right)^2 |I_1|^2. \tag{4}$$

The radiation resistance is therefore

$$R_1 = \frac{2}{3}\frac{R_0}{4\pi}\left(\frac{2\pi l_1}{\lambda}\right)^2 = 20\left(\frac{2\pi l_1}{\lambda}\right)^2. \tag{5}$$

In this formula, the length of the dipole is expressed as a fraction of the radianlength ($2\pi l_1/\lambda$).

Four such small dipoles may form the basis for computing the radiation resistance of a small square loop (of area $A_1 = l_1^2$). In a direction parallel to one pair of sides, only the other pair radiate and they nearly cancel each other. The residual far field is $2\pi l_1/\lambda$ of that of one side because this is the angle of the difference of their distance and retardation. The directive pattern is that of a small magnetic dipole which, like the small electric dipole, fills $\frac{2}{3}$ of the sphere in space. Therefore the radiation resistance of the loop is that of one side, multiplied by the power ratio $(2\pi l_1/\lambda)^2$.

$$R_1 = \frac{2}{3}\frac{R_0}{4\pi}\left(\frac{2\pi l_1}{\lambda}\right)^4 = 20\left(\frac{2\pi l_1}{\lambda}\right)^4$$
$$= 20\left[\frac{A_1}{(\lambda/2\pi)^2}\right]^2. \tag{6}$$

The strength of the equivalent magnetic dipole is proportional to the area (A_1). If there are several parallel turns carrying the same current (I_1), the effective area is the total area of all turns.

Efficiency in Terms of Interception Area

The available-power efficiency (if small) is simply formulated from the radiation quantities:

$$\frac{P_2}{P_1} = \frac{|Z_{12}|^2}{4R_1R_2}. \tag{7}$$

Substituting for these quantities in terms of length dimensions, and generalizing each R by changing from $\frac{2}{3}$ to $1/g$:

$$\frac{P_2}{P_1} = \frac{1}{4} g_1 g_2 \left(\frac{\lambda}{2\pi r}\right)^2 = g_1 g_2 \frac{\pi(\lambda/2\pi)^2}{4\pi r^2}$$
$$= g_1 g_2 \frac{\text{area of radian circle}}{\text{area of distance sphere}}. \tag{8}$$

The last two forms were discovered by the writer [2]; the first form was published by Friis [4].

Fig. 2 illustrates this rule for the basic simple case of two isotropes, while Fig. 3 does the same for the more general case, exemplified by two small dipoles.

Since sending and receiving are reciprocal functions, it is natural to identify the interception area of each one. This is diagramed in Fig. 3, showing the area each presents to the other. Letting this area be $A = g\pi(\lambda/2\pi)^2 = g$ radiancircles, for each antenna, the efficiency becomes [2], [4]:

$$\frac{P_2}{P_1} = \frac{A_1 A_2}{(4\pi r^2)\pi(\lambda/2\pi)^2} = \frac{A_1 A_2}{r^2\lambda^2}$$
$$= \frac{(\text{sending area})(\text{receiving area})}{(\text{distance sphere})(\text{radiancircle})}. \tag{9}$$

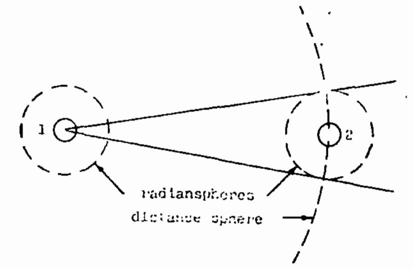

Fig. 2—Area of interception for two isotropes.

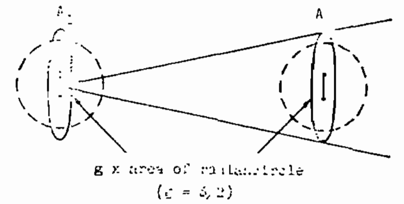

Fig. 3—Area of interception for two small dipoles.

By the simple formulas, two isotropes at a distance of one radianlength have a coupling efficiency of $\frac{1}{4}$. This is approximately valid, being in the transition between high and low efficiency. In general, this occurs at a distance of $\sqrt{g_1 g_2}$ radianlengths, by (8). At lesser distances, the interaction complicates the formula for efficiency.

The mutual impedance may be expressed in terms of the values of radiation resistance by rearranging (7) and (8).

$$|Z_{12}| = 2\sqrt{R_1 R_2}\sqrt{P_2/P_1} = \frac{\lambda}{2\pi r}\sqrt{g_1 g_2 R_1 R_2}. \qquad (10)$$

This is a corollary to the theorem of interception area [2]. It was independently discovered by Huntoon at NBS during the war while studying the problem of proximity fuzes [3].

The receiving antenna reradiates an amount of power equal to the available power it delivers to the matched load. If instead the antenna is tuned without adding any resistance, the received current is doubled. The second antenna then reradiates four times its available power. This rule is limited to a small antenna.

A large flat array with a reflector can be designed to intercept all the power incident on its area. Its power ratio of directivity, by comparison with the isotope, is then

$$g = \frac{\text{area}}{\pi(\lambda/2\pi)^2} = \frac{4\pi(\text{area})}{\lambda^2} = \frac{\text{area}}{\text{radiancircle}}. \qquad (11)$$

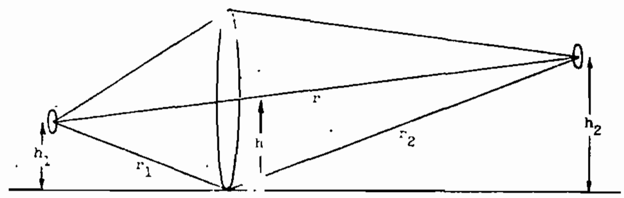

Fig. 4—Intermediate area of interception for two antennas over plane ground.

If the area is covered by many dipoles pitched $\frac{1}{2}$ wavelength in a rectangular array, the power ratio of directivity is seen to be π times the number of dipoles.

Two antennas may be located above the ground at heights so low that there is near-cancellation of direct and reflected waves. Fig. 4 shows the geometry of such a case. It is assumed that the path difference is less than one radianlength ($h_1 h_2 < r\lambda/4\pi$) and that the ground is a flat surface with a reflection coefficient of minus one (which is typical of imperfect conductors near grazing incidence). It can be shown that

$$|Z_{12}| = \frac{2h_1 h_2}{r^2}\sqrt{g_1 g_2 R_1 R_2}; \qquad \frac{P_2}{P_1} = g_1 g_2\left(\frac{h_1 h_2}{r^2}\right)^2. \qquad (12)$$

Taking the partial distances as shown,

$$r_1 = r\frac{h_1}{h_1 + h_2}; \qquad r_2 = r\frac{h_2}{h_1 + h_2}. \qquad (13)$$

The intermediate height (h) is that of the direct line over the point of reflection:

$$h = \frac{2h_1 h_2}{h_1 + h_2}; \qquad 1/h = \tfrac{1}{2}(1/h_1 + 1/h_2). \qquad (14)$$

This is taken as the radius of an intermediate circular area. It is found that the transmission efficiency is the product of two values, one computed by (9) from the first antenna to the intermediate circle, and the other from this circle to the second antenna. The proximity of the ground has the effect of an intermediate aperture as shown.

All Components of Mutual Impedance

In Fig. 1, the complex mutual impedance of two small dipoles has three terms at distances much greater than the dimension of the dipoles but not necessarily greater than the radianlength. These components are readily derived from the formula for the transverse electric field given in textbooks:

$$Z_{12} = \frac{R_0}{4\pi}\,\frac{j2\pi l_1}{\lambda}\,\frac{j2\pi l_2}{\lambda}\left[\left(\frac{2}{j2\pi r}\right)^3 + \left(\frac{\lambda}{j2\pi r}\right)^2 + \left(\frac{\lambda}{j2\pi r}\right)\right]\exp - \frac{j2\pi r}{\lambda}. \qquad (15)$$

(ohms) (sphere) — (length angles) — (C) — (R) (distance angle) — (L) — (retard)

The coefficient in front of the brackets [ ] is equal to $-\frac{3}{2}\sqrt{R_1R_2}$, in terms of radiation resistance. In Ramo-Whinnery [9] is found an expression which emphasizes the significance of the three components (C, R, L); this expression is revised as follows to give the three components the dimensions of impedance:

$$Z_{12} = \frac{l_1 l_2}{4\pi r^2}(1/j\omega\epsilon_0 r + R_0 + j\omega\mu_0 r)\exp - j2\pi r/\lambda. \quad (16)$$

This form is instructive and is also useful for evaluating the equivalent circuit elements (C, R, L). The preceding form (15) is the ultimate in dimensional simplicity.

Fig. 5 shows the network equivalent to two small dipoles, giving a breakdown of the three components of mutual impedance, and their variation with distance (r). They are marked (*) to denote that they are subject to retardation with distance.

Fig. 6 shows the variation of the three components with distance. At a distance of one radianlength, the three components are equal in magnitude, so that first and third cancel, leaving only the resistance. At lesser distances, the capacitive coupling predominates; at greater distances, the inductive coupling predominates, as derived above for the far field.

In any other direction, these components are modified. The far component disappears if either dipole is in line with the radial distance.

Spherical Small Antenna

In relation to a spherical wave and the radiansphere, the ideal shape of a small antenna might be spherical. There is one such antenna that is significant. It is a "magnetic dipole" simulated by a spherical inductor [10], [12].

Fig. 7 shows such an inductor. Its winding is pitched uniformly in the axial direction. Its core may be filled with magnetic material (k_m).

If the length of wire is much less than the resonant length, the magnetic field inside is uniform, and outside has the same pattern as that of a small magnetic dipole. (Such an inductor is mentioned by Maxwell but is seldom found in the more recent literature; the writer made use of this concept about 1941.)

The inductance of this sphere is

$$L = \frac{2\pi}{3}\mu_0 a n^2 \frac{1}{1 + 2/k_m}. \quad (17)$$

Its radiation resistance is

$$R = \frac{2\pi}{3}R_0 n^2 \left(\frac{2\pi a}{\lambda}\right)^4 \left(\frac{1}{1 + 2/k_m}\right)^2. \quad (18)$$

Its inductive reactance is

$$\omega L = \frac{2\pi}{3}R_0 n^2 \frac{2\pi a}{\lambda}\frac{1}{1 + 2/k_m}. \quad (19)$$

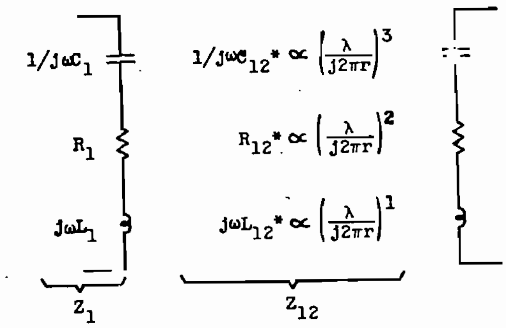

Fig. 5—Network equivalent to two small electric dipoles.

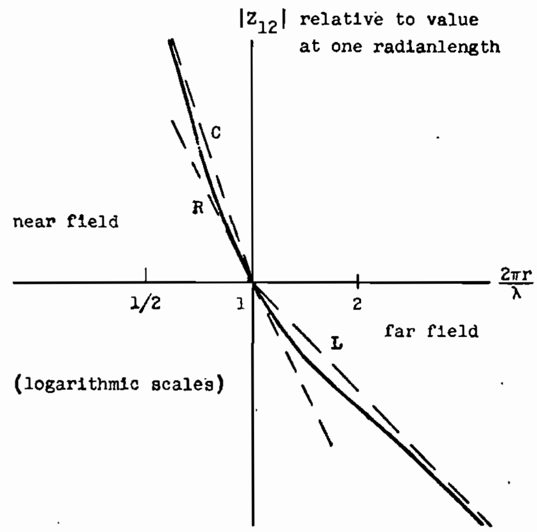

Fig. 6—Variation of components of mutual impedance.

Therefore the radiation power factor is

$$p = R/\omega L = \left(\frac{2\pi a}{\lambda}\right)^3 \frac{1}{1 + 2/k_m}$$

$$= \frac{\text{volume of inductor sphere}}{\text{volume of radiansphere}} \frac{1}{1 + 2/k_m}. \quad (20)$$

This relation reaches the ultimate simplicity for the ideal case of a perfect magnetic core ($k_m = \infty$) so that there is no stored energy inside the coil. This limiting case is represented in Fig. 8.

The radiansphere may be regarded as a hypothetical inductor whose internal energy is the stored energy of the magnetic field, and whose external energy is the radiating power. The small antenna radiates by virtue of its coefficient of coupling with the radiansphere; the above ratio (20) is proportional to the square of this coefficient of coupling.

Practical Small Antennas

In a previous paper, the writer has treated the topic of practical small antennas [5]. Special emphasis was placed on the role of the volume occupied by the antenna in determining its radiation power factor. A cylindrical volume was taken as a basis for comparing electric dipoles with magnetic dipoles (air-core coils).

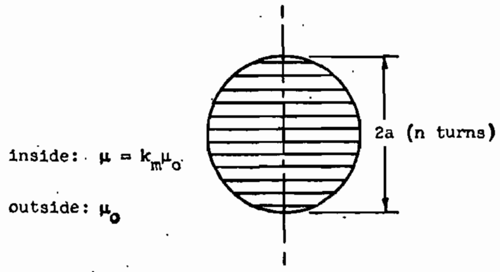

Fig. 7—Spherical inductor.

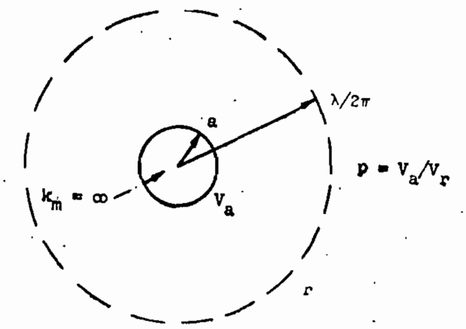

Fig. 8—Spherical inductor in radiansphere.

Some of the most common forms of small antennas are of such shape that the occupied volume is no longer significant. This is true of an electric dipole made of a straight wire or rod or tower. It is also true of a magnetic dipole made of a thin loop of wire. Either of these would require a certain size of sphere to contain it. Since this volume is only partially utilized, it is presumptive that the antenna would have a radiation power factor much less than the theoretical upper limit for this size of sphere.

The opposite extreme is a flat capacitor with air-dielectric or a long inductor with air core. In these cases, the radiation power factor approaches a lower limit of 2/9 the value for an ideal sphere (with perfect magnetic core) of the same volume.

It is interesting to compare two small antennas of opposite kinds occupying the same circular-cylindrical space, namely the disk capacitor (C) and the solenoid inductor (L), each having an air core. The shape chosen is a cylinder of equal diameter and height. As compared with an ideal sphere of the same volume (having no energy stored inside), each kind has a power factor less than the ideal by the following factors:

$$C: \quad \frac{8}{\pi}\,\frac{2}{9} = 0.57$$

$$L: \quad \left(1 + \frac{4}{3\pi}\right)\frac{2}{9} = 0.32.$$

In this rating, the power factor of the capacitor is about twice that of the inductor. However, the latter can be increased by a factor of two or more by inserting an iron core, while the former cannot be increased by any known materials.

The comparison with air cores brings out a basic difference between the two kinds. The external (useful) stored energy of the capacitor is about $\frac{2}{3}$ of the total, while that of the inductor is about $\frac{1}{3}$. This is because the inside and outside flux paths differ in impedance in a ratio of about two to one; these paths are in parallel for the capacitor and in series for the inductor. Decreasing the effective length of internal flux path by inserting some material has the effect of increasing the stored energy in the capacitor but decreasing it in the inductor. The latter is advantageous.

If a small antenna is restricted in its maximum dimension but not in its occupied volume, the radiation power factor is increased by utilizing as much as possible of the volume of a sphere whose diameter is equal to this dimension. The cylinder discussed above is a good practical compromise. The practical limitations of capacitor and inductor are only slightly different, so the choice may be determined by other considerations (such as wave polarization, loss power factor, associated circuits, construction, and environment).

A special case is a small antenna operating underground or underwater. These mediums are dissipative toward a electric field but not a magnetic field. Therefore the loop antenna is much to be preferred for efficiency of radiation in either of these environments [11].

Radiation Shield

For purposes of measurement, it may be desired to remove the radiation resistance of a small antenna while retaining its other properties (loss resistance, capacitance, inductance). This can be accomplished to a close approximation by enclosing the antenna in a radiation shield which ideally is a perfectly conducting spherical shell whose inner surface is located at the radiansphere. (See Fig. 8, for example.) This prevents the radiation while causing little disturbance of the near field. In practice, the size, shape, and material are not critical. A cylinder with one or both ends open may suffice.

The writer devised this test for a very small loop antenna operating at a frequency such that the radiansphere had a convenient size; the loop was in an oscillating circuit so the radiation shield caused an increase in the amplitude of oscillation. The increase in amplitude was a measure of the radiation efficiency. In general, the radiation shield enables the separate determination of loss resistance and radiation resistance.

Conclusion

The radiansphere around a small antenna is logically regarded as the boundary between the near field of stored energy and the far field of radiating power. There is not a definite boundary but rather a transition, since the terms associated with the near field predominate inside and those associated with the far field predominate outside. The interception area defined for the hypothetical isotropic antenna is the area of the radiancircle, a projection of the radiansphere, so the latter is logically regarded as the sphere of influence of such an antenna. An idealized small spherical antenna is found to have a radiation power factor equal to the ratio of its volume over that of the radiansphere. A radiation shield is described whose ideal location is at the radiansphere. All of these concepts are helpful in visualizing and remembering the rules governing small antennas, especially their near field and far field.

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Radio-Wave Propagation in the Earth's Crust[1,2]

Harold A. Wheeler

(February 26, 1960)

There is a reasonable basis for postulating the existence of a useful waveguide deep in the earth's crust, of the order of 2 to 20 km below the surface. Its dielectric is basement rock of very low conductivity. Its upper boundary is formed by the conductive layers near the surface. Its lower boundary is formed by a high-temperature conductive layer far below the surface, termed the "thermal ionosphere" by analogy to the well-known "radiation ionosphere" far above the surface.

The electrical conductivity of the basement rock has not been explored. An example based on reasonable estimates indicates that transmission at 1.5 kc/s might be possible for a distance of the order of 1500 km.

This waveguide is located under land and sea over the entire surface of the earth. It may be useful for radio transmission from the shore to a submarine on the floor of the ocean. The sending antenna might be a long conductor in a drill hole deep in the basement rock; the receiving antenna might be a vertical loop in the water.

In the earth's crust, there appears to be a deep waveguide that has not yet been explored. This waveguide extends under all the surface area, so it suggests the possibility of wave propagation under the ocean floor. This might enable communication from land to a submarine located on or near the ocean floor. If below a certain depth, it happens that the excess radio noise from electric storms would become weaker than thermal noise, and no other source of appreciable radio noise is recognized.

This waveguide comprises basement rock as a dielectric between upper and lower conductive boundaries. The upper boundary is formed of the well-known geological strata located between the surface and the basement rock, with conductivity provided by electrolytic solutions and semiconductive minerals. The lower boundary is provided by high-temperature conductivity in the basement rock.

In concept, the lower boundary is similar to the usual ionosphere, being formed by gradually increasing conductivity. In the usual ionosphere [Wait, 1957], caused by extraterrestrial radiation, the conductivity increases with height. In the present case, however, the conductivity increases with depth and is caused by the increasing temperature in the dielectric material. Therefore it may be designated as the "inverted ionosphere" or "thermal ionosphere."

Figure 1 shows how this waveguide may be used for communication from a shore sending station (S) to an underwater receiving station (R). The latter may be a submarine on the bottom of the ocean. The sender launches a vertically polarized transverse-electro-magnetic (TEM) wave by means of a vertical wire projecting into the basement rock. The wave is propagated in the deep waveguide between the surface conductor and the thermal ionosphere. Some power from the wave leaks out of the waveguide into the ocean just above, and is sampled by an antenna at the receiver.

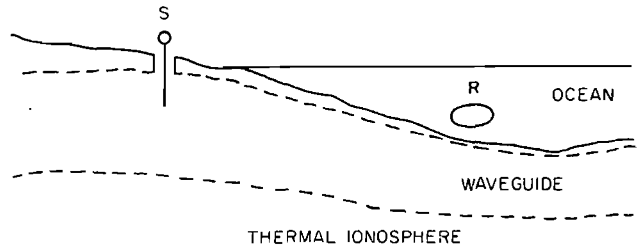

FIGURE 1. *Communication through the deep waveguide under the ocean.*

Figure 2 shows an arrangement for the sender antenna. It is a long conductor (pipe) sunk into a drill hole filled with oil insulation. The example shown has conducting material down to a depth of about 1 km. Through this layer of earth, there is an outer pipe which forms the outer conductor of a coaxial transmission line. This pipe is in contact with a conducting surface such as water or radial wires in the ground. Below this layer of earth, the inner conductor extends further about 2 km into the basement-rock dielectric. This extension radiates into the waveguide in the usual manner.

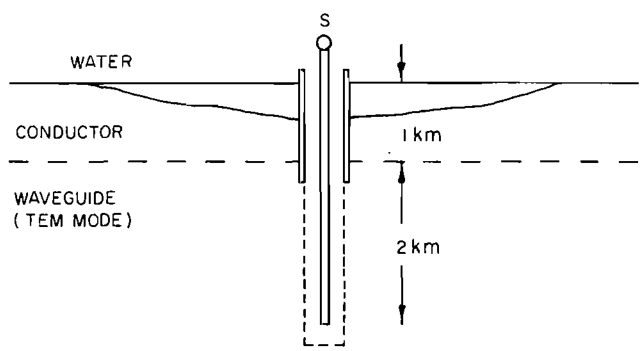

FIGURE 2. *Sender antenna in the deep waveguide.*

[1] Contribution from Wheeler Laboratories, Great Neck, N.Y., and Developmental Engineering Corp., Leesburg, Va.
[2] Paper presented at Conference on the Propagation of ELF Radio Waves, Boulder, Colo., January 27, 1960.

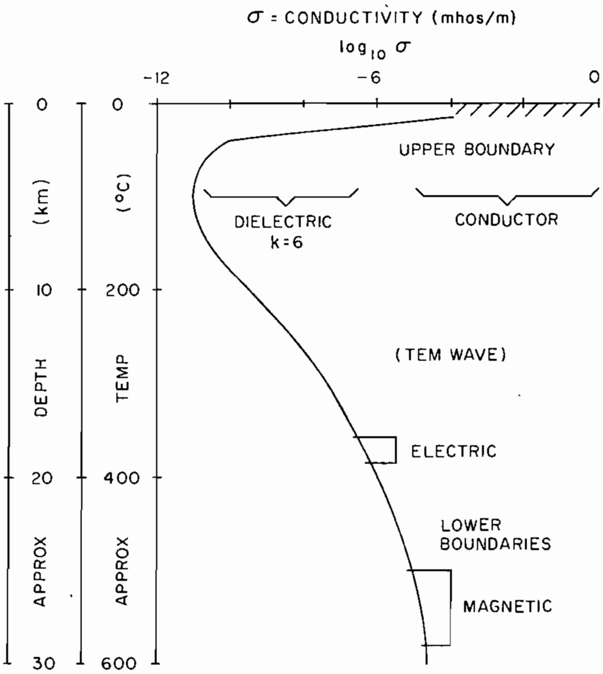

FIGURE 3. *Variation of conductivity with depth to form the deep waveguide.*

Figure 3 shows how the temperature and the resulting conductivity may vary with depth, especially in the basement rock at depths exceeding a few kilometers. This diagram will be used to explain the expected behavior of the deep waveguide. At depths of about 2 to 20 km, the basement rock is indicated to have such low conductivity that it is a dielectric suitable for wave propagation. Above and below this dielectric, the conductivity is high enough to serve the function of boundaries for the waveguide.

The upper boundary is fairly well defined, in a depth of the order of 1 km (perhaps down to several kilometers). Its conductivity, in the most common materials, ranges from a maximum of 4 mhos/m in sea water down to about 10^{-4} in rather dry nonconductive minerals.

The dielectric layer, shown between depths of about 2 and 20 km, may have very low conductivity, of the order of 10^{-6} to 10^{-11} mho/m. The lowest conductivity is observed in fused quartz, but probably is not found in nature. The present plan is useful if the conductivity is around 10^{-8} or lower. The dielectric constant is about 6.

The lower boundary has some unusual properties. (These are also characteristic of the ionosphere at frequencies below VLF.) The gradual increase of conductivity [Van Hippel, 1954] provides an effective boundary for each kind of field, that for the electric field being closer than that for the magnetic field. In each case, there is a sort of skin depth in the boundary [Wheeler, 1952]. Both of these boundaries make comparable contributions to the total dissipation factor of the waveguide, which determines the exponential attenuation rate.

The location of each boundary depends on the frequency, the conductivity, and the rate of change of conductivity with depth. In the example to outlined, these boundaries occur at temperatures ... the range of 300 to 600 °C.

As an example of the behavior that might be expected in this waveguide, the following numerical values are suggested.

Frequency	1.5 kc/s
Dielectric constant	6
Wavelength (in dielectric)	80 km
Effective boundaries of E field (depths)	1–18 km
Effective boundaries of M field (depths)	1–27 km
Skin depth for E field (lower boundary)	1.5 km
Skin depth for M field (lower boundary)	4 km
Length of radiator (in waveguide)	2 km
Reactance of radiator	1600 ohms
Effective length of radiator	1 km
Radiation resistance (in waveguide)	0.4 ohms
Other resistance	20 ohms
Radiation efficiency	0.02
Average power factor of E and M fields, about	0.1
Napier distance (for wave attenuation)	130 km
Decibel distance (for wave attenuation)	15 km
100–db distance	1500 km

If these values are to be experienced, communication ranges of the order of 1500 km will be possible under the surface of the earth.

The assumptions for this example are based on preliminary estimates of the best conditions that are at all likely to be realized. The high-temperature conductivity needed for the lower boundary is typical of quartz and other similar minerals. The extremely low conductivity at lower temperatures is unlikel-- but need not be quite so low to provide a dielect that could give the indicated performance.

As for the properties of the basement rock, it is very doubtful how low its conductivity may be. Its seismic properties are explored but not its electrical conductivity. Its principal chemical components are known, but apparently not its small content of "impurities" that may determine the conductivity. It seems that core samples have been made to only a small depth (less than 1 km) in the basement rock, presumably because there has been little prospect of valuable mineral products at a reasonable cost. It is notable that some tests show a trend toward lower conductivity (below 10^{-6}) in the transition from the surface layers into the basement rock. A continuation of this trend may enable such performance as is indicated in the example.

Returning to the waveguide properties, the TEM mode (with vertical polarization) is the one that has the greatest probability of enabling long-range communication. It is the only propagating mode at frequencies below about 2 kc/s (including the above example).

This preliminary study has indicated that the deep waveguide is probably a definite physical phenomenon. The properties of its dielectric and boundaries are not known quantitatively, so it is uncertain to what distances this waveguide may be useful for communication or related purposes. Some rather optimistic assumptions as to these propertie^s^

lead one to speculate on distances of the order of 1500 km. While the deep waveguide extends under the entire surface of land and sea, it is most needed for radio transmission to a submarine on the ocean floor, because this location is shielded from the usual radio waves above the surface.

This concept occurred to the writer recently during discussions with Lester H. Carr and his associates in Developmental Engineering Corporation, notably L. E. Rawls, G. F. Leidorf, and their geological consultant, P. Parker. The opportunity of working with this group is acknowledged with appreciation.

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(Paper 65D2–119)

JOURNAL OF RESEARCH of the National Bureau of Standards—D. Radio Propagation
Vol. 65D, No. 2, March–April 1961

Publications of the National Bureau of Standards*

Selected Abstracts

Use of the incoherent scatter technique to obtain ionospheric temperatures, T. E. VanZandt and K. L. Bowles, *J. Geophys. Research* **65,** *No. 9, 2627–2628 (Sept. 1960).*
If the ion-electron gas is in diffusive equilibrium on the topside of the *F* layer, then the electron density decreases exponentially with height, and its logarithmic decrement is proportional to the neutral gas temperature. From an electron density profile obtained by the scatter radar technique, it is shown that this interpretation is consistent. Moreover, the deduction of ionospheric temperatures in this way from scatter radar electron density profiles has several advantages over other methods.

Correlation of an auroral arc and a subvisible monochromatic 6300 A Arc with outer-zone radiation on November 28, 1959, B. J. O'Brien, J. A. VanAllen, F. E. Roach, and C. W. Gartlein, *J. Geophys. Research* **65,** *No. 9, 2759–2766 (Sept. 1960).*
During a severe geomagnetic storm on November 28, 1959, two Geiger tubes on satellite Explorer VII (1959 iota) found anomalies in the outer radiation zone at an altitude of about 1000 km which appear to be correlated in space and time with optical emissions from the atmosphere beneath. Very intense narrow zones of radiation were detected over a visible aurora during one pass. The radiation in three such zones was harder toward low latitudes. On three subsequent passes the radiation zone was deduced to be over a subvisible 6300 A arc, whose brightness diminished as the radiation zones became less intense. The correlation is discussed.

Seasonal variations in the twilight enhancement of [OI] 5577, L. R. Megill, P. M. Jamnick, and J. E. Cruz, *J. Atmospheric and Terrest. Phys.* **18,** *No. 4, 309–314 (Aug. 1960).*
Measurements of the twilight enhancement of [OI] 5577 were obtained during the period September 1957 to December 1958 at Rapid City, S.D. All these measurements were normalized to the intensity at sunset or sunrise at a height of 100 km. The results obtained indicate that there was a seasonal dependence of the twilight enhancement of [OI] 5577 emission. The enhancement occurred most frequently in the autumn and winter months, the maximum occurring about 1 November. The enhancement almost never occurred during the spring and summer months.

Some magnetoionic phenomena of the Arctic *E*-region, J. W. Wright, *J. Atmospheric and Terrest. Phys.* **18,** *No. 4, 276–289 (Aug. 1960).*
Several unusual phenomena of *E*-region ionogram echoes obtained at Thule, Greenland (mag. dip 85.5°) are described. They are explained as the effects of electron collisions on the propagation of radio waves at high-magnetic latitudes. The third magnetoionic component (Z-echo) is explained in this way and several of its distinguishing features are explained and illustrated. New phenomena demonstrate the existence of an *E*-pause (valley above $h_{max}\,E$), and permit the measurement of electron densities and collision frequencies therein.

Widely separated clocks with microsecond synchronization and independent distribution systems, T. L. Davis and R. H. Doherty, *IRE Wescon Conv. Record* **4,** *pt. 5, 3–17 (1960).*
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The timing system also includes a unique method for distribution of several time code formats on a single UHF channel.

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Other NBS Publications

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VLF Propagation Under the Ionosphere in the Lowest Mode of Horizontal Polarization

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In the lower part of the VLF band, around 4 kc/s, it appears that the lowest rate of attenuation is obtainable by horizontal polarization in the TE–01 mode. This offers a substantial advantage relative to vertical polarization in the usual TM–01 mode and the simple TM–00 or TEM mode. Some types of antennas are found to be suitable for the TE–01 mode, namely, a horizontal wire above ground or a vertical loop, either one located in a plane perpendicular to the direction of transmission. A theoretical study is summarized, leading to the conclusion that this mode offers some unique features and is suitable for transmission to distances of the order of 4,000 km.

1. Introduction

The practical utilization of the VLF band has been limited to vertical polarization, and the same is true of most theoretical studies. It now appears that there is a part of the frequency spectrum which would be best utilized by the use of horizontal polarization.

Practical utilization of the VLF band (3 to 30 kc/s) has been limited to the upper part (10 to 30 kc/s) where admittedly the most efficient transmission under the ionosphere appears to be obtained by vertical polarization. The prospects for the lower part (3 to 10 kc/s) have been dimmed by the observation of maximum attenuation of noise (at 3 to 4 kc/s) but it has not been sufficiently emphasized that this handicap applies only to vertical polarization.

The appreciation of this fact about 4 years ago encouraged the writer to consider the alternative of horizontal polarization for utilization of frequencies in the lower part of the VLF band (say around 4 kc/s). The resulting theoretical studies indicate the utility of this part of the spectrum is best utilized by horizontal polarization.

It is found that a single mode of HP, the TE–01 mode, promises the lowest rate of attenuation of all modes, in a limited range of the spectrum. The principal advantage is the more effective reflection from the ionosphere. Another advantage is the use of a single mode as distinguished from a mixture of the two principal modes of vertical polarization, the usual TM–01 and the simple TM–00 or TEM.

The use of a different mode naturally introduces opportunities and problems that are challenging. These will be presented and summarized in support of the thesis. The essential characteristics of different types of antennas will be compared. An example (computed for 4 kc/s) indicates that useful transmission is possible to distances of the order of 4,000 km.

2. Theory of Propagation

Propagation with horizontal polarization under the ionosphere has some major differences from vertical polarization, in regard to its behavior and limitations. Therefore it is essential to have a clear view of these distiuctions.

In order to present the subject in proper perspective, it will be necessary to compare the horizontally polarized mode with its closest competitors, the lowest two of the vertically polarized modes. These different modes are designated as follows:

Vertical polarization,	TM–00 or TEM mode.
Vertical polarization,	TM–01 mode.
Horizontal polarization,	TE–01 mode.

These designations follow the pattern of mode numbers in a rectangular waveguide. The first digit (0) denotes uniform field across the width, which results from the absence of side walls. The second digit (0 or 1) denotes the number of "half-wave" cycles of field variation in the height of the waveguide. There is a recognizable correlation between these designations and the same or similar ones in use by other writers.

Figure 1 shows the field pattern of the subject mode, TE–01, characterized by horizontal polarization of the electric field (E). The upper and lower boundaries of the waveguide are the ionosphere and the ground (land or water), separated by the effective height (h). The magnetic field (H) has a vertical component and also has a component in the direction of propagation, which are related to the concept of two component waves with oblique propagation. Each "half-wave" cycle of the field pattern is associated with a half-wavelength in the guide ($\frac{1}{2}\lambda_g$).

The principal discussion of the waveguide behavior will be based on magnetic-dipole (loop) antennas at the sending and receiving ends (S and R). The vertical loop is the one kind of antenna which is

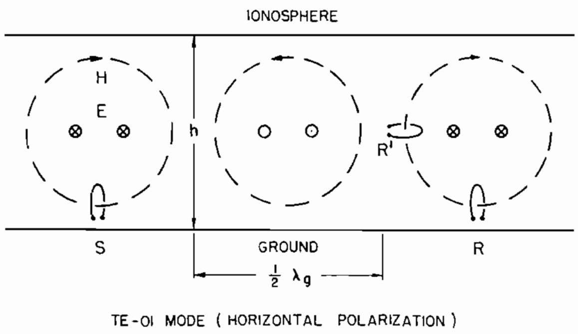

FIGURE 1. *Wave propagation in TE–01 mode under ionosphere.*

adapted to all three modes. Its operation in "vertical polarization" is well known, in that the transverse magnetic field (TM) is horizontal. Its operation in horizontal polarization is unusual, in that the horizontal longitudinal component of magnetic field is the principal component near the boundaries (ionosphere and ground). Therefore the loop axis is along the line of propagation, a direction which gives no radiation in free space. Here it does couple with the waveguide mode. This may be explained by its substantial radiation in the oblique directions of propagation of the pair of component waves in the guide.

Figure 1 shows how vertical loops in coaxial relation may be used on the ground for sending and receiving (S and R). On aircraft at a substantial altitude, the horizontal component of magnetic field decreases while the vertical component increases (respectively reaching zero and maixmum at $\frac{1}{2}h$). Therefore a horizontal loop on an aircraft may serve as a receiving antenna (R').

By comparison with figure 1, the vertically polarized modes have different patterns with only horizontal magnetic field. The TM–00 or TEM mode has uniform vertical electric field terminating on the boundaries. The TM–01 mode has a vertical component of electric field of maximum value but opposite polarity at the upper and lower boundaries. Also it has a horizontal component in the direction of propagation, which is not usually utilized.

For this study, the following ionosphere conditions are assumed; they are believed to be typical of the actual range of conditions:

Effective height of ionosphere:	$h=75$ km.
Cutoff wavelength of 01 modes:	$\lambda_c=2h=150$ km.
Cutoff frequency:	$f_c=2$ kc/s.

These conditions are based on the concept that the boundaries behave as fairly good conductors.

This concept fails for the TM–01 mode under usual VLF operating conditions (10 to 30 kc/s); this will be taken into account in drawing conclusions. Under the extreme conditions of low conductivity and high frequency, the upper boundary behaves more like a magnetic wall (open-circuit instead of short-circuit), the phase of reflection being reversed. This concept would lead to a mode designated TM–0½ and a nominal cutoff frequency of 1 kc/s. However, the latter is not realized, because this concept is not applicable near cutoff.

As mentioned above, the waveguide behavior can be explained in terms of a pair of oblique waves reflected from the boundaries. Figure 2 shows this familiar concept as applied to the present subject, particularly the TE–01 and TM–01 modes. The sending antenna, with its images in the boundaries, forms a pair of waves that are incident on the boundaries at an elevation angle (ψ) from parallel (grazing incidence). These waves undergo reflection at the boundaries. At the ionosphere, the reflection coefficient (ρ) is less than unity and this deficiency causes a loss in propagation out to the transmission radius (r). Corresponding loss in the ground reflection (not shown) is much less and will be ignored. The propagation loss may be computed from the reflection loss multiplied by the number of reflections.

The ionosphere may be described in terms of its conductivity (σ) and some resulting derived factors, as follows:

$$\text{Dissipation factor: } p=\frac{\sigma}{\omega\epsilon}=2\left(\frac{\lambda}{2\pi\delta}\right)^2 \tag{1}$$

$$\text{Skin depth: } \delta=\sqrt{\frac{\lambda}{\pi\sigma R_c}};$$

$$\frac{2\pi\delta}{\lambda}=\sqrt{\frac{4\pi}{\lambda\sigma R_c}}=\sqrt{\frac{1}{30\sigma\lambda}}=\sqrt{2/p} \tag{2}$$

in which

σ=conductivity, uniform above a certain level of altitude (mhos/meter).
$\omega=2\pi f$=radian frequency (radians/second).
$\epsilon=8.85\times10^{-12}$=electrivity (electric permittivity) in free space, including ionosphere (farads/meter).
λ=free-space wavelength corresponding to the operating frequency (meters).
$R_c=377$=free-space wave resistance across a square area of wave front (ohms).
δ=skin depth on surface of conductor (meters).

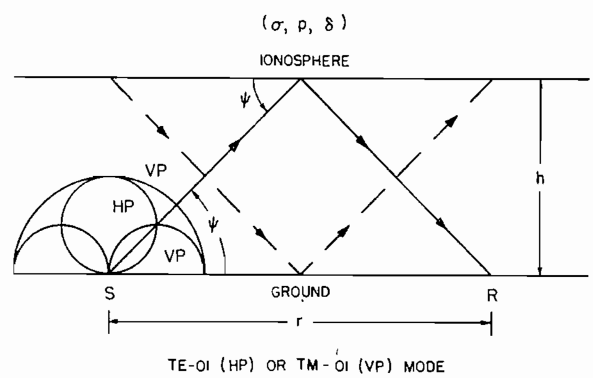

FIGURE 2. *Theory of reflection for evaluation of transmission efficiency.*

In these formulas, the ionosphere is taken to have "pure conductivity," free of electricity; hence they are valid only if the displacement currents are much less than the conduction currents at any frequency under consideration.

It is noted that the ionosphere has a gradual transition of conductivity, rather than the definite boundary here assumed. This factor will be discussed further on.

Figure 2 shows incidentally the relations between the radiation patterns of a ground antenna (S) and its coupling with the waveguide mode. These patterns are cross sections of the familiar "doughnut" pattern in various orientations. Each of these patterns is obtainable by a well-known simple antenna, such as a vertical wire, a raised horizontal wire, or a vertical loop with proper orientation.

The HP pattern is that of a vertical (axial) loop or a raised horizontal (transverse) wire. It shows zero radiation in the direction from S to R. However, it shows substantial (less than maximum) radiation in the oblique direction of propagation of a component wave in the guide. Therefore it gives substantial coupling between the antenna and the TE–01 mode of horizontal polarization.

The elevation angle (ψ) of oblique propagation in the waveguide is defined as follows:

$$\sin\psi=\lambda/\lambda_c;\ \cos\psi=\lambda/\lambda_g;\ \tan\psi=\lambda_g/\lambda_c \qquad (3)$$

in which λ_g is the so-called "guide wavelength."

As previously mentioned, the attenuation in propagation may be computed from the reflection loss, as determined by the elevation angle and the conductivity. This concept fails for the TM–00 mode, in which the elevation angle is zero, so it is not adequate for comparison of the three modes under consideration.

We return to the familiar viewpoint of waveguide modes, between conductive boundaries giving nearly complete reflection. The resulting attenuation can be computed from simple formulas, to be given. The relative attenuation in the three modes is graphed in figure 3. The simplest mode (TM–00) is taken as a reference, because it propagates over the entire frequency range.

The following formulas give the attenuation in each of the three modes for the present configuration, in the limiting case of nearly complete reflection at the walls [Marcuvitz, 1951],

$$\text{TM–00 (VP)}:\ \alpha_0=\frac{1}{4\pi}\frac{2\pi\delta}{\lambda_c}\frac{2\pi r}{\lambda}. \qquad (4)$$

Here the height (h) is replaced by ½ wavelength at cutoff ($\frac{1}{2}\lambda_c$) for the other two modes, although this mode has no cutoff.

$$\text{TM–01 (VP)}:\ \alpha_v=\frac{1}{2\pi}\frac{2\pi\delta}{\lambda_c}\frac{2\pi r}{\lambda}\frac{\lambda_g}{\lambda} \qquad (5)$$

$$\alpha_v/\alpha_0=2/\cos\psi \qquad (6)$$

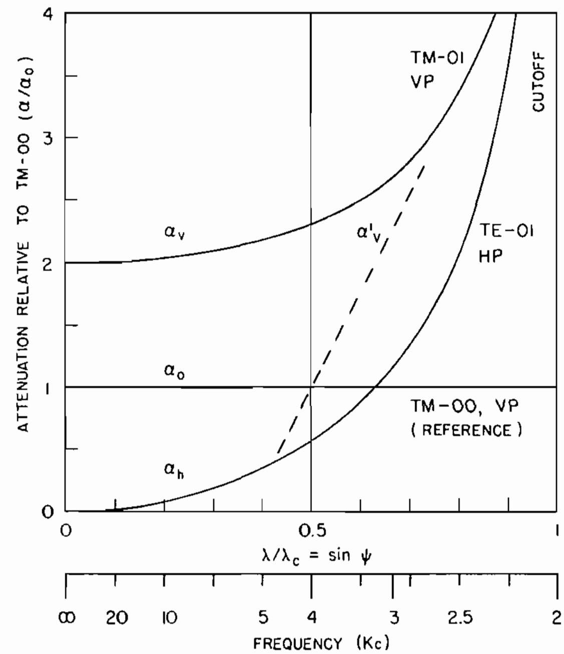

FIGURE 3. *Relative attenuation of propagation in modes TM-00, TM-01, and TE-01.*

$$\text{TE–01 (HP)}:\ \alpha_h=\frac{1}{2\pi}\frac{2\pi\delta}{\lambda_c}\frac{2\pi r}{\lambda_c}\frac{\lambda_g}{\lambda_c} \qquad (7)$$

$$\alpha_h/\alpha_0=2\sin^2\psi/\cos\psi=2\sin\psi\tan\psi \qquad (8)$$

$$\alpha_h/\alpha_v=(\lambda/\lambda_c)^2=\sin^2\psi. \qquad (9)$$ [2]

In these formulas α is the attenuation (napiers) for the radial distance (r).

It is noted that this attenuation is only the component caused by dissipation in the upper boundary. This is added to the basic attenuation caused by radial divergence of the wave from sender to receiver. From the mode viewpoint in the present situation, this is a cylindrical wave guided between parallel plane conductors.

Referring to figure 3, the horizontal scale is the wavelength ratio (λ/λ_c) which is inversely proportional to frequency; this is chosen to present the entire range of propagation on a finite scale. Also it shows clearly the limiting conditions far from cutoff.

The two modes associated with oblique propagation have attenuation increasing toward the cutoff frequency. This may be explained by the increasing number of reflections in covering the distance.

The dissipation is caused by magnetic field at the upper boundary. Far from cutoff, the HP mode has the maximum concentration of magnetic field midway between the boundaries, so the dissipation is much the least of the three modes. This is analogous to the familiar TE–01 mode in circular waveguide. As in that case, the TE–01 mode between parallel planes has the least attenuation of all modes, far

[2] This equation (9) is in agreement with a formula of J. R. Wait [1960].

from cutoff. The marginal condition is

$$\alpha_h/\alpha_0=1 : \lambda/\lambda_c=\sin\psi=\lambda/\lambda_c=\sqrt{\frac{2}{\sqrt{17}+1}}=0.625;$$

$$f/f_c=\lambda_c/\lambda=1.60. \quad (10)$$

For the ionosphere, these relations are complicated by the fact that it may not behave as a "pure conductor," especially as it affects the TM–01 mode. The conduction current may be comparable with the displacement current. Therefore there is some difficulty in evaluating the apparent conductivity in the simple model here assumed.

In practical conditions, the curves in figure 3 fall short of representing the relative attenuation in the several modes. This happens from the failure of two assumptions: first, that the reflecting boundary is a good conductor; and second, that it has a step transition between the conductor and the wave medium. The former is to be discussed here; the latter at a point further on.

Continuing on the assumption of a step boundary, the relation shown between TM–00 and TE–01 modes is qualitatively valid for the usual conductivity of the ionosphere, at frequencies not too far from cutoff. On the other hand, the curve for the TM–01 mode (α_v) fails entirely for reasons associated with the angle of minimum reflection (Brewster angle). Far from cutoff (beyond the Brewster angle), at small angles approaching grazing incidence, the ionosphere behaves more like a magnetic wall (open-circuit) than an electric wall (short-circuit) for reasons not to be elaborated here. As a result, the waveguide propagation behaves more like the TM–0½ mode. The ionosphere becomes a much better reflector, so the attenuation becomes the least of all modes, approaching ¼ the attenuation of the TE–01 mode. The transition to this phenomenon is qualitatively indicated by the dashed line (α_v').

The location of the dashed line in figure 3 is indefinite because the relative attenuation varies with conditions. There is an experimental basis for specifying its intersection with unity, or the attenuation crossover of the TM–01 and TM–00 modes. It happens that VP measurements of atmospheric noise show a definite minimum at a certain frequency [C.C.I.R., 1957; Jean et al., 1961; Maxwell et al., 1963] and it may be inferred that one of these modes is predominant on either side of this frequency. The frequency of minimum noise is centered at 3 to 4 kc/s, presumably nearer the higher value for conditions of lower ionosphere (height near 70 km). Therefore 4 kc/s is here taken as a typical value of the crossover frequency, and the dashed line in figure 3 is so drawn.

From these considerations, figure 3 shows that the HP mode may have the least attenuation of all modes in the frequency range of about 3 to 5 kc/s. At least, it is competitive with the two VP modes. This in itself is interesting and may not have been stated in earlier publications. However, there are other factors that give it a unique advantage, as will be explained.

For a stable transmission over a long range from one point to another, it is preferable to utilize only one waveguide mode. Figure 3 shows the two modes with vertical polarization, and furthermore shows that they are equal in attenuation at some frequency near 4 kc/s. Diurnal variation of the ionosphere height will cause a variation of several cycles in their relative phase at the receiver. Therefore we have the possibility of slow periodic fading and occasional deep fading, in contrast to stable transmission.

On the other hand, there is only one mode with horizontal polarization and maximum amplitude. In the range of 3 to 5 kc/s, we may consider the utilization of this TE–01 mode to the exclusion of others. It has comparable attenuation, perhaps the lowest, and there would be no opportunity for fading. Therefore it may be the most useful mode for this frequency range. This concept is the subject of this paper. It will be considered further with reference to the practical problems of utilization.

The evaluation of attenuation in terms of skin depth (δ) is based on the concept of a wall of uniform high conductivity with a definite plane boundary. The gradual boundary of the ionosphere is better approximated by an exponential profile of conductivity, under study by the writer during the past 4 years [Wheeler, 1959, 1960, 1961, 1962] and more intensively by others [Wait and Walters, 1963]. There is a difference in behavior that can be described simply for horizontal polarization but not so simply for vertical polarization. The reasons for this will be given briefly.

In the usual "skin effect," the "hard" boundary of a good conductor acts as the boundary for both E and M fields. It causes appreciable loss associated with the M field but no appreciable loss associated with the E field. On the other hand, the "soft" boundary of the ionosphere may cause its effective level to appear at different heights, lower for the E field than for the M field [Wheeler, 1959, 1960, 1961]. This contributes a loss also associated with the E field, which is comparable with that of the M field. In the actual profile of the ionosphere, the lower level of the E-field boundary is usually in a region of more gradual variation, so the E-field loss is likely to be the greater of the two components.

With vertical polarization, both E and M fields are maximum at a conductive boundary, so both are subject to the losses of a "soft" boundary. With horizontal polarization, however, the M field is maximum and the E field is minimum at a conductive boundary, so only the M-field losses are appreciable. This enables a simple description of the boundary reflection for horizontal polarization [Wait and Walters, 1963].

In the TE–01 mode, the boundary losses can be computed by making the following substitution in formula (7). The effective skin depth appears to be

$$\delta=\pi h_1 \quad (11)$$

in which h_1 is the "napier height" in the exponential profile, that is, the change of height in which the conductivity increases in the ratio of e, the base of natural logarithms. (This rule was developed by writer in 1960.) In other words,

$$\frac{\text{conductivity at } h+\Delta h}{\text{conductivity at } h}=\exp \Delta h/h_1. \qquad (12)$$

In the usual skin effect, the skin depth is proportional to the ½ power of wavelength ($\delta \propto \sqrt{\lambda}$). On the other hand, in the exponential profile, the apparent skin depth is invariant with wavelength. This implies that the reflection takes place at different levels of conductivity, proportional to wavelength, so the usual variation with wavelength is compensated.

Referring to figure 3, if the skin depth is constant, the attenuation in the reference mode TM–00 varies inversely with wavelength or directly with frequency. Either side of the crossover of the two VP modes, one or the other has decreasing attenuation. This is consistent with the concept that the VP noise spectrum is subject to maximum attenuation at the crossover.

In figure 3, the extra E-field loss in VP is ignored. It is expected that the relative attenuation of the HP mode will be found to be much less, perhaps down to ⅓ the amount indicated by the curves. If so, this will yield a great advantage for HP over VP in the frequency range shown around 4 kc/s. The attenuation rate for the VP modes has been reviewed recently [Maxwell et al., 1963] and the resulting values in the frequency range are much greater than would be expected for HP [Wait and Walters, 1963].

Here we should be reminded that the VLF sending station has to compete with a high level of atmospheric noise from lightning. There is a need for tests of the HP noise spectrum, since only the VP spectrum has been reported. Horizontal lightning strokes are presumably more frequent than vertical, and have a height advantage over an HP antenna near the ground. They are not expected to have any minimum of spectral density, since the crossover phenomenon is avoided in HP. These factors suggest that HP may be subject to a handicap in the noise level to be surmounted by radiation of power from the sending antenna.

It has been assumed that higher modes have much greater attenuation, and therefore are relatively too weak to be useful or to cause appreciable cancellation. The justification is very simple. We may compare the "01" mode with the "0N" mode of higher order ($N>1$). First we note that the cutoff wavelength is $1/N$ as great, so the angle of elevation (ψ) becomes about N times as great; this causes about N times the loss in one reflection. (The reflection loss approaches zero at grazing incidence.) Second, this means about N times the number of reflections for the same distance. We conclude that the rate of attenuation in the "0N" mode is about N^2 times as great as that in the "01" mode. This comparison applies similarly to the TM and TE modes in the present discussion.

With vertical polarization, the imperfect conductivity of the ground contributes appreciable attenuation, greater at higher frequency. This effect is severe in case of dry ground, especially mountains or ice cap. On the other hand, with horizontal polarization, this component of attenuation is less in general; in particular it is less at higher frequency and is negligible in comparison with the ionosphere component.

For any particular mode in a waveguide, there is a coupling factor associated with any type of antenna. This coupling may be evaluated as complete (1) or partial but useful (<1) or nominally zero (0). In these terms, table 1 gives the rating of various types of antennas with respect to the three modes of interest.

TABLE 1. *Mode coupling of various types of antennas*

Antenna (at ground)	Mode coupling		
	VP TM–00	VP TM–01	HP TE–01
Omnidirective:			
(1) Vertical wire	1	<1	0
(2) Horizontal wire, crossed pair	0	<1	<1
(3) Vertical loop, crossed pair	1	1	<1
Directive:			
(4) Oblique wire, end direction	<1	0	0
(5) Horizontal wire, end direction	0	<1	0
(6) Horizontal wire, side direction	0	0	<1
(7) Vertical loop, planar direction	1	1	0
(8) Vertical loop, axial direction	0	0	<1

NOTES:
(2)(3) Rotary phase; can use diversity with opposite rotation.
(4) Designed to cancel TM–01 mode in one direction at one frequency.
(4)(5)(6)(8) Selection of single mode, effective in only one direction.
(1) and (2) Sending-receiving combination for selection of TM–01 mode, both omnidirective.
(2) and (7) Sending-receiving combination for selection of TM–01 mode, one omnidirective and the other directive; can use (7) for direction finding.
(1) and (7) Sending-receiving combination for both VP modes, the usual combination; can use (7) for direction finding.

Figure 4 gives some curves of the variation of mode coupling with wavelength or frequency. As in figure 3, these are referred to the TM–00 mode, and the same horizontal scale is used. These are based on the vertical loop antenna, being the only type that couples with all three modes.

The following formulas give the mode coupling ratios on the assumption that the antenna is a small horizontal magnetic dipole (vertical loop) located on or near the lower boundary (ground).

$$\frac{\text{TM–01 (VP)}}{\text{TM–00 (VP)}}: k_v=\sqrt{\frac{2}{\cos\psi}} \qquad (13)$$

$$\frac{\text{TE–01 (HP)}}{\text{TM–00 (VP)}}: k_h=\sin\psi\sqrt{\frac{2}{\cos\psi}}=\sqrt{2\sin\psi\tan\psi} \qquad (14)$$

in which k is the mode-coupling voltage or current ratio referred to the TM–00 (TEM) mode.

This ratio may be evaluated for a receiving loop, by comparing the values of induced voltage from plane waves of two modes, carrying equal values of power. In general, the mode coupling depends

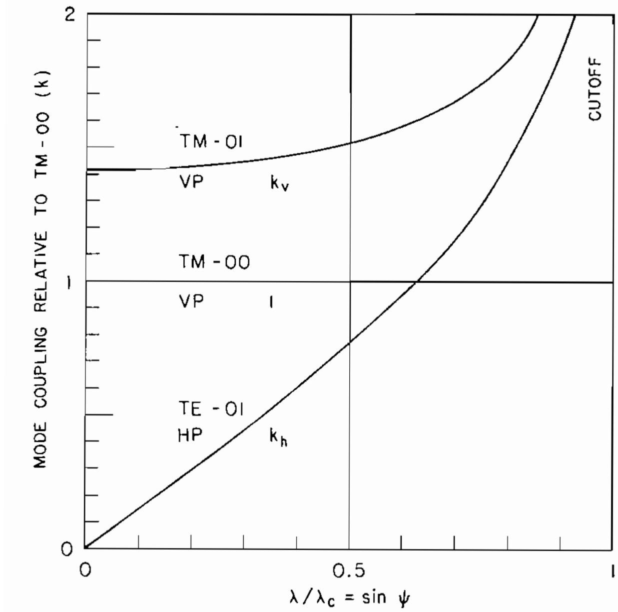

FIGURE 4. *Relative mode coupling between loop and modes TM–00, TM–01, and TE–01.*

on the factors shown in figure 2, namely, the elevation angle (ψ) and the antenna radiation pattern. For VP, the vertical loop has a pattern invariant with angle, so the coupling (k_v) includes only the waveguide effect (in terms of cos ψ). For HP, the pattern contributes an extra factor (sin ψ), which is of particular interest in the present discussion.

The mode coupling ratios are simply related with the attenuation ratios, as will be seen by comparing (6) and (8) with (13) and (14). This results from the fact that they both depend on the magnetic field adjacent to the boundaries.

In figure 4, the mode coupling ratio (k_v) between the two VP modes is nearly constant, except near cutoff, and is not far from unity. On the other hand, the HP mode coupling (k_h) varies more than proportional to wavelength. This is because the coupling increases with the elevation angle (ψ), as previously mentioned with reference to figure 2.

In the range of 3 to 5 kc/s, of particular significance in figure 3, the coupling of the HP mode is somewhat less than that of the VP modes. However, this slight deficiency is present only at each end of the circuit; over a long distance, it is easily offset by a slightly lower rate of attenuation, or by other advantages such as stability against fading.

Referring to figure 4, the mode coupling of the loop may be compared with that of a straight wire. A vertical wire couples only with the VP modes; the coupling with TM–01 decreases toward zero at cutoff. A horizontal wire couples with TM–01 and TE–01 modes, not with TM–00; its coupling with TM–01 or TE–01 has the same variation as k_h in figure 4.

Referring to table 1, there are tabulated the mode-coupling ratings of eight types of antennas that may be interesting with respect either to omnidirective coverage or to unique properties in one directio The table is self-explanatory, so the discussion w be limited to some relations of particular interest.

The vertical wire is the only antenna that couples with only VP, not HP in any direction. The horizontal wire or the vertical loop couples with both VP and HP; the loop couples with all three modes. These two types have directive properties that enable the selection of either VP or HP, to the exclusion of the other, in some direction.

It is concluded that the single-mode feature of HP can be assured only in a point-to-point circuit, having one or both antennas of type (6) or (8). Each of these couples only HP in one direction.

3. Antennas

The utilization of any particular mode of propagation requires the selection of types of antennas which are suited for coupling with the mode. In the present discussion, the use of horizontal polarization instead of the usual vertical polarization requires coupling with the TE–01 mode instead of the usual TM–01 or TM–00 modes. The introduction to the mode in figure 1 shows the common loop antenna used in different orientations relative to the direction of transmission from sender (S) to receiver (R or R').

Figure 5 shows the horizontal wire (a) and the vertical loop (b), which are alternative types for utilizing the single-mode feature of HP propagatio·· in a direction perpendicular to the vertical pla of the antenna. Each will be described, for the case of a "small" antenna, having effective length (l) and effective height (l') less than ⅛ wavelength. The simple structures to be shown are typical of sending antennas of rather large size for radiation of high power.

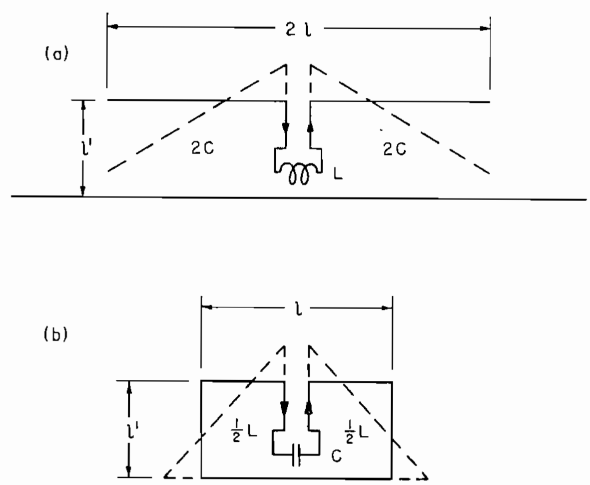

FIGURE 5. *Alternative sending antennas on the ground.*

(a) Horizontal electric dipole (horizontal wire).
(b) Alternative sending antennas on the ground.

Figure 5(a) shows a horizontal wire which behaves a horizontal electric dipole of a certain effective length (l) and effective height (l'). It has a balanced coupling with the circuit, so the effective capacitance (C) is that of the two halves in series. It is tuned to resonance by an inductor (L).

Figure 5(b) shows a vertical loop which behaves as a horizontal magnetic dipole of a certain effective area ($l \times l'$). It has a balanced coupling with the circuit, so the effective inductance (L) is that of the two halves in series. It is tuned to resonance by a capacitor (C).

In figure 5(a) or 5(b), the HP mode coupling is determined by the effective area ($l \times l'$). The top wire may be horizontal (solid lines) or sloping (dashed lines). The latter offers a particular advantage for the horizontal wire (a) because the current is greater near the center; as a result, the center has to be raised only half as much as the ends are lowered, so the average height can be decreased. In either case, the sloping form enables the use of a single tower.

For omnidirective coverage, either type can be utilized in a crossed pair having quadrature relative phase of coupling with the circuit. The sloping form enables the use of a single tower for both antennas of the pair.

As in any capacitive antenna, the use of grounded metal supporting structures causes a reduction of effective height of the horizontal wire (a). In an inductive antenna, such as the loop (b), a thin vertical tower has no appreciable effect; the use of a thick metal tower or grounded guy wires may cause reduction of effective height, but less than the effect on a capacitive antenna.

As in "small" sending antennas in general, the radiation of high power requires a higher voltage on the capacitive type (a) and a higher current on the inductive type (b). This essential difference and other practical considerations influence the choice between these two types [Wheeler, 1958a].

A receiving antenna may be much smaller, for two reasons. First, there is no requirement for handling high power. Second, a rather small size is sufficient to receive atmospheric noise at a level higher than circuit noise. There is more freedom of design in the smaller size.

A small electric dipole has two practical limitations. First, the reactance becomes so high as to impose difficulties in circuit design, and there is no way of transforming the reactance to a lower value in the antenna. Second, there is no core material available for enabling a further reduction in size while retaining the same efficiency.

A small magnetic dipole or loop offers advantages in both of these respects. First, while the reactance of one turn is low, it may be transformed to a convenient value by using a number of turns. Second, a ferrite core enables a further reduction in size while retaining the same efficiency. These and other practical considerations lead to the use of a small loop for reception, especially for submarine reception while submerged [Wheeler, 1958b].

The radiation resistance of an antenna under the ionosphere is essentially different from that in free space. It is seldom formulated, because there is no practical difference in the usual VLF band (10 to 30 kc/s), such that the ionosphere is more than two wavelengths from the antenna.

At lower frequencies, nearer to cutoff in the waveguide between ionosphere and ground, there is a need for restating the radiation resistance. The ionosphere has more effect on the resistance. Furthermore, the power radiated in each mode can be separately identified with a value of radiation resistance. This enables an evaluation of the efficiency of radiation into the mode to be utilized.

The resistance representing radiation into the TE-01 mode, in either of the types shown in figure 5, is given by the following formula, derived from familiar waveguide principles,

$$R=\frac{\pi}{4} R_c \left(\frac{2\pi l}{\lambda}\right)^2 \left(\frac{l'}{h}\right)^2 \frac{\lambda_g}{\lambda_c}=\frac{1}{4\pi} R_c \left(\frac{2\pi l}{\lambda_c}\right)^2 \left(\frac{2\pi l'}{\lambda_c}\right)^2 \frac{\lambda_c}{\lambda}\frac{\lambda_g}{\lambda}$$

$$=\frac{1}{2\pi} R_c \left(\frac{2\pi l}{\lambda_c}\right)^2 \left(\frac{2\pi l'}{\lambda_c}\right)^2 \frac{1}{\sin 2\psi}. \tag{15}$$

The variation with frequency becomes very simple. There is a minimum value for the condition,

$$f/f_c=\sqrt{2};\ \psi=\pi/4;\ \sin 2\psi=1. \tag{16}$$

In the waveguide modes under the ionosphere, its effective height is involved directly (h) or indirectly in terms of the resulting cutoff wavelength (λ_c). This cylindrical wave is an essential distinction from the spherical wave in free space, especially in the usual cases where one mode predominates over the others.

The efficiency of radiation into this mode is the ratio of R over the total resistance. The latter includes not only the heat losses in the antenna and ground and the rest of the tuned circuit, but also the radiation into any other propagating modes. High efficiency of radiation into one mode requires the exclusion of coupling with any other propagating modes.

The antennas of figure 5 couple not only with the TE-01 mode of HP propagation, but also with one or both of the VP modes (in other directions). Therefore a high efficiency of radiation in the TE-01 mode is impossible. In practice, however, the heat losses are usually sufficient to preclude high efficiency, so the extra radiation resistance of other modes is a minor handicap.

4. Transmission Efficiency

The transmission circuit from sender to receiver should be evaluated with reference to all significant factors. Some of these are elusive but generally recognized so they need not be elaborated here. A few that are particularly relevant or unusual will be described with reference to a computed example.

Table 2 outlines a set of conditions and various factors resulting therefrom. It is based on a point-to-point transmission circuit relying on horizontal polarization in the TE–01 mode under the ionosphere. The natural conditions are given values that are typical to the extent of present knowledge based directly or indirectly on experimental evidence. The operating conditions are selected to bring out the capabilities of horizontal polarization. The table is self-explanatory, as a basis for some comments.

The frequency (4 kc/s) is approximately that at which horizontal polarization appears to offer the greatest advantage over vertical polarization. It is double the cutoff frequency (2 kc/s) based on the ionosphere height.

The attenuation by radial divergence of a cylindrical wave is given by the power ratio,

$$\frac{\lambda/2\pi}{2\pi r}. \qquad (17)$$

No attempt is made to refine this ratio in respect to the guide wavelength and any directivity that may be utilized.

The least is known about the height interval of transition from dielectric to conductor in the ionosphere serving as the upper boundary of a waveguide. This is expressed in terms of the "napier height" related to the most recent publication of Wait and et al. [1963]. The value assumed (2 km) is said by him to be typical of daytime ionosphere, in a discussion directed to horizontal and vertical polarization at frequencies in the VLF band. The reflection coefficient is computed from his formula, and is related to the effective skin depth as discussed herein. This leads to the attenuation caused by loss in reflection at the ionosphere.

TABLE 2. *Transmission circuit using horizontal polarization*

Waveguide mode	TE–01
Frequency	$f=4$ kc/s
Wavelength	$\lambda=75$ km
Cutoff frequency	$f_c=2$ kc/s
Cutoff wavelength	$\lambda_c=150$ km
Ionosphere height	$h=\frac{1}{2}\lambda_c=75$ km
Napier height	$h_1=2$ km
Effective skin depth	$\delta=\pi h_1=6.3$ km
Effective conductivity	$\sigma=1.6$ $p\mu-$mho/m
Effective dissipation factor	$p=7.2$
Mode coupling factor	$k_h=0.76$
Elevation angle	$\psi=\pi/6=30°$
Ionosphere reflection coefficient	$\rho_h=0.59=-4.6$ db
Distance per ionosphere reflection	$2h/\sin\psi=300$ km
Distance	$r=4000$ km
Attenuation:	
Radial divergence	30 db
Ionosphere reflection	60±8 (day/night)
Mode coupling (both ends)	3
	93±8 db

The diurnal variation of attenuation is even more uncertain. The value given is estimated on the basis of some experimental evidence for vertical polarization [Maxwell et al., 1963] and modified for the expected improvement in stability for horizontal polarization.

The loss in mode coupling at both ends of the circuit is evaluated for the TE–01 mode relative to the TM–00. It becomes a minor factor in this example.

The three kinds of attenuation included in this outline emphasize the peculiarities of this mode. It is notable that the major factor is the attenuation caused by incomplete reflection from the ionosphere, even though this is much less for horizontal polarization. These components of attenuation add up to about 100 db for a distance of 4,000 km. This is typical of the greatest distance over which the TE–01 mode may be useful.

An implicit factor in reaching this conclusion is the noise level from lightning. This is an unknown for horizontal polarization, but the receiving antenna responds also to waves of vertical polarization arriving from other directions. The rate of attenuation in both polarizations discriminates against noise sources at great distances, so the noise level far from the equator is much lower than usually experienced in the active VLF range of 10 to 30 kc/s.

5. Conclusion

It appears that there is a part of the frequency spectrum that may be best utilized in the TE–01 waveguide mode under the ionosphere, characterized by horizontal polarization. Some of the advantages and other peculiarities of this mode may be summarized as follows.

(1) In a frequency range near 4 kc/s, this mode promises the lowest rate of attenuation of all modes under the ionosphere, as in the familiar TE–01 mode in circular waveguide. In comparison with its closest competitors, the TM–00 and TM–01 modes characterized by vertical polarization, it is expected to offer about half the rate of attenuation. Its cutoff frequency is near 2 kc/s, and its advantage appears at somewhat higher frequencies.

(2) The mechanism of reflection at the ionosphere is simpler for horizontal polarization, involving mainly the magnetic field rather than both electric and magnetic fields. It is expected that there will be less variation of attenuation with changing conditions (diurnal cycles, disturbances, etc.). This advantage is particularly noticeable in the theory and operation of the "soft" boundary prevalent below the ionosphere, as distinguished from the "hard" boundary previously assumed as a simple model.

(3) There are some antenna configurations suited for coupling with the TE–01 mode. The coupling is somewhat less than that available for vertical polarization, but this small handicap is negligible in comparison with the gain by the lower rate of attenuation.

(4) In a point-to-point transmission circuit, there are simple types of antennas that select HP modes to the exclusion of VP modes. Since the TE–01 mode has much the lowest attenuation of all HP modes, this feature can be used to assure single-mode propagation. This is the most stable condition in a waveguide because there is no possibility of fading caused by interference between two modes. C

or both of the antennas should be a horizontal wire above the ground or a vertical loop, in a plane perpendicular to the direction of the transmission path. The former is particularly suited for the sending end, the latter for the receiving end.

(5) The loss on reflection from the ground is the least for horizontal polarization, being much less than the loss on reflection from the ionosphere. Therefore there is no appreciable handicap in transmission over ground of low conductivity, such as mountains or ice cap.

(6) The noise level for HP has not been thoroughly measured and reported. In the subject frequency range, the moderately high attenuation reduces the contribution from noise sources at a long distance. This is particularly true of VP noise, to which any receiving antenna is likely to be responsive in some directions.

(7) It happens that there is no simple antenna, for location just above the ground, which will select HP to the exclusion of VP in all directions. Therefore we cannot secure omnidirective coverage in both transmission and reception with the full advantage of HP. This is not an appreciable handicap at any frequency for which the HP mode has much the lowest rate of attenuation. This factor does preclude the use of HP for accurate direction finding, because a loop antenna would be confused by a mixture of HP and VP. Perhaps this last property could be used intentionally to confuse the direction finding attempts of an enemy.

(8) The use of frequencies near 4 kc/s, in contrast the lowest frequencies now in use (14–20 kc/s), would reduce the attenuation rate in sea water to about one-half. This is an obvious advantage in communication to a submerged submarine, but must be weighed in the context of the other significant factors.

A computed example indicates that the subject mode should be useful for communication over a distance of the order of 4,000 km.

It is submitted that this mode offers the most effective utilization of a certain part of the frequency spectrum, in transmission under the ionosphere. Therefore further theoretical and experimental studies are recommended.

The present understanding of mode propagation under the ionosphere is attributed to J. R. Wait, and has been relied on as the background for this presentation. It is an application of the waveguide principles that are familiar in the field of microwaves, as most intensively developed at the M.I.T. Radiation Laboratory during World War II.

To the National Bureau of Standards, and particularly to the Staff at Boulder, Colo., we are all most deeply indebted for their studies of radio noise. Here we have relied on the more specialized tests of the noise spectrum, reported by A. D. Watt, E. L. Maxwell, A. G. Jean, and their associates.

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